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Determination of Task Temporal Variations in Construction Scheduling Using Imprecise Probability

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CONSTRUCTION SCHEDULING

- In construction project management, scheduling and monitoring is crucial in successful and timely completion of project.
- Scheduling is performed through objective evaluations of each task involved in both spatial (physical correlation) and temporal (sequential correlation) domains.
- Temporal aspect of each task is defined by its expected duration, intended start time, and intended completion time.
- This duration is function of numerous factors including logistics, finance, and environmental conditions
- Because of those project circumstances, duration of each task may experience variations and uncertainties.

UNCERTAINTIES IN CONSTRUCTION SCHEDULING

- In order to guarantee that overall project completion time is not significantly exceeded, these uncertainties must be addressed so the allowable temporal variation for each task can be quantified.
- However, deterministic schemes are not capable of quantification of those uncertainties.
- Moreover, the traditional probabilistic approaches require a significant level of available data to correctly obtain the probability distribution function for each task.
- As the available data is limited, these approaches may result in errors when estimating those variations.

CURRENT METHODS FOR CONSTRUCTION SCHEDULING

The Critical Path Method (CPM)

- Utilized for planning and monitoring of both industrial and construction projects.
- Determines the overall duration of a project.
- Identifies, as well, the project's critical tasks whose temporal variations may increase and affect the overall project duration.
- Analysis performed in conventional CPM is deterministic.
- Does not consider the uncertainties in the activities' durations.

CURRENT METHODS FOR CONSTRUCTION SCHEDULING (Cont.)

Program Evaluation and Review Technique (PERT)

- Considers the presence of uncertainty in using probability theories.
- Task durations are described in terms of three values, the minimum, expected and maximum durations (used to define mean and standard deviation for each task).
- Requires a significant level of accuracy in the selection of the Probability Density Function (PDF).
- Assumes, mostly, a Beta distribution for each activity which may exhibit complexities regarding the notion of skewness.

POSSIBILISTIC APPROACH – FUZZY

- In the fuzzy approach to scheduling, the durations of activities are defined as fuzzy variables with predefined membership functions

- Previous works:

  Lessmann, Muhlogger, and Oberguggenberger (1994)
  Netzplantechnik mit unscharfen Methoden

  Fetz, Oberguggenberger, Jager, Koll, Krenn, Lessmann, and Stark (1999)
  Fuzzy Models in Geotechnical Engineering and Construction Management

  Lu and Abourizk (2000)
  Simplified CPM/PERT simulation model

ALTERNATE APPROACH – IMPRECISE PROBABILITY

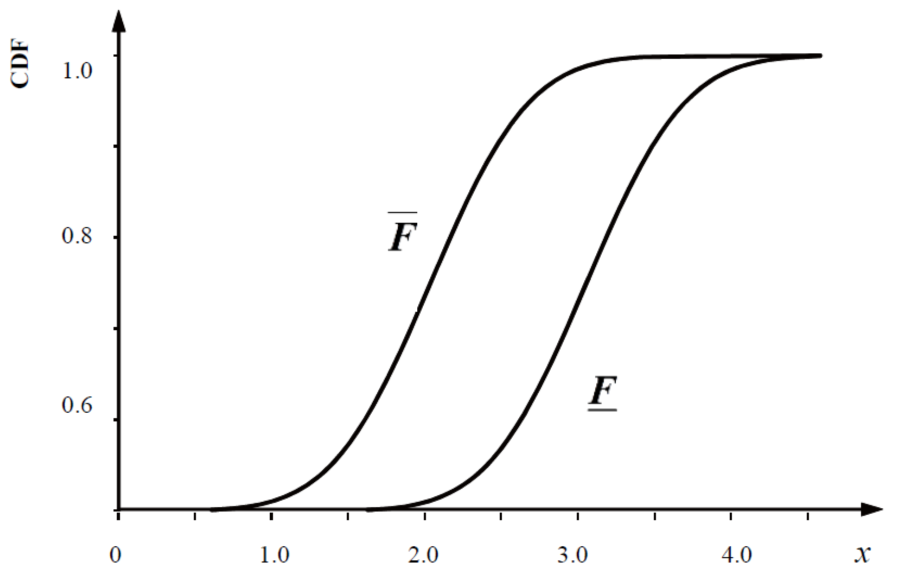
- An alternative to having sufficient data to construct the “correct’ distribution is to estimate a range of distributions that represent possible activity duration.
- Imprecise probability theory can handle uncertainty in a system with no assumptions of a well-defined PDF.
- Imprecise probability theory addresses the extraction of useful information about a process in the case that the PDF of system inputs or parameters may not be known.

RESEARCH OBJECTIVE

To develop a more robust methodology for construction planning and scheduling with uncertainties quantified through imprecise probability concepts.

IMPRECISE PROBABILITY

- Framework for handling incomplete information with uncertain PDF or CDF
- Involves setting bounds on CDF based on deterministic or non-deterministic parameters (mean, variance, etc.)



COMBINATION IN IMPRECISE PROBABILITY STRUCTURES

- Dempster-Shafer Structures
- Dependency Bounds Convolutions (Williamson and Downs 1990)
- Probability Bounds Analysis (Ferson and Donald 1998)
- Many methods expanded by Ferson et al. 2003, including: Enveloping
- The developed methodology uses enveloping for combining p-boxes

METHODOLOGY

IMPRECISE PROBABILITY PROJECT SCHEDULING (IPPS)

I. Determine the imprecise probability structure, based on P-boxes, for each activity

For each activity:

a) Construct an independent imprecise probability structure for uncertain duration
b) Construct a P-box structure for the uncertain duration

II. Perform forward pass and backward pass analyses

Performed using the constructed P-box structures for the uncertain duration of each activity (using enveloping)

METHODOLOGY

IMPRECISE PROBABILITY PROJECT SCHEDULING (IPPS)

In Step II, for each activity:

Forward Pass: Using P-box addition, determine the P-box structures of:

1) Early start time
2) Early finish time

Backward Pass: Using P-box subtraction, determine the P-box structures of:

1) Lag time(s) associated with the activity
2) Free float time
3) Total float time
4) Late start time
5) Late finish time

METHODOLOGY

IMPRECISE PROBABILITY PROJECT SCHEDULING (IPPS)

III. Identify the critical path(s)

As the path(s) from the start through the end of the network where every activity has zero total float.

IV. Identify the P-box structure for the total project duration

For the entire analysis, a probability level be chosen for indexing purposes

IPPS COMPUTATIONS PROCEDURE

Interval-based computations for P-box structures are performed for all combinations of corresponding intervals.

For one combination and considering two P-box structures $\boldsymbol{X}$ and $\boldsymbol{Y}$ with intervals $\boldsymbol{X_i} = [\underline{x_i}, \overline{x_i}]$ and $\boldsymbol{Y_j} = [\underline{y_j}, \overline{y_j}]$ corresponding to their i and j discretization levels:

In the forward path for addition:

$$\boldsymbol{X_i} + \boldsymbol{Y_j} = [\underline{x_i}, \overline{x_i}] + [\underline{y_j}, \overline{y_j}] = [\underline{x_i} + \underline{y_j}, \overline{x_i} + \overline{y_j}]$$

In the backward path for subtraction :

$$\boldsymbol{X_i} - \boldsymbol{Y_j} = [\underline{x_i}, \overline{x_i}] - [\underline{y_j}, \overline{y_j}] = [\underline{x_i} - \overline{y_j}, \overline{x_i} - \underline{y_j}]$$

IPPS INVERSE PROBLEM DEPENDENCY

As the backward path is an inverse problem of the forward path, the input and output interval variables are dependent. Direct application of interval subtraction may yield results with overestimation.

To reduce this overestimation, the interval variables are defined based on their midpoints and radii, as ($\boldsymbol{\varepsilon} = [-1,1]$):

$$\boldsymbol{X_i} = mid(\boldsymbol{X_i}) + rad(\boldsymbol{X_i}) \times \boldsymbol{\varepsilon}$$

$$\boldsymbol{Y_j} = mid(\boldsymbol{Y_j}) + rad(\boldsymbol{Y_j}) \times \boldsymbol{\varepsilon}$$

Therefore, the interval subtraction equation is:

$$\boldsymbol{X_i} - \boldsymbol{Y_j} = mid(\boldsymbol{X_i}) - mid(\boldsymbol{Y_j}) + \left(rad(\boldsymbol{X_i}) - rad(\boldsymbol{Y_j})\right) \times \boldsymbol{\varepsilon}$$

The overestimation is reduced because of the single representation of $\boldsymbol{\varepsilon}$.

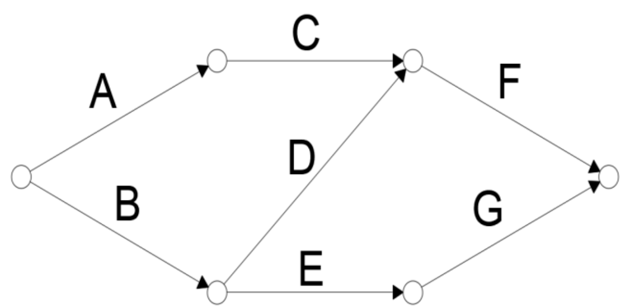
NUMERICAL EXAMPLE

Simple Network (Chu 2008)

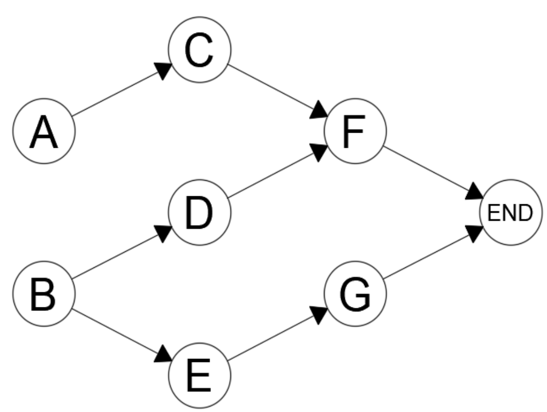
Task	Min Duration	Expected Duration	Max Duration	Preceding Task(s)
A	7	10	16	-
B	4	5	6	-
C	26	30	50	A
D	35	40	58	B
E	8	12	14	B
F	30	36	48	C and D
G	8	9	12	E

GRAPHIC ILLUSTRATION

Tasks on Links (Chu 2008)



Tasks on Nodes (Solution)



SOLUTION

- CPM: The duration of each task is the expected duration

$$Dur_i = expected_i$$

- PERT: The duration of each task is the PERT mean

$$Dur_i = \mu_{PERT_i}$$

- IPPS: The duration of each task is a normal distribution with mean and standard deviation equal to the PERT mean and PERT standard deviation

$$Dur_i = N(\mu_{PERT_i}, \sigma_{PERT_i})$$

- Interval Analysis: The duration of each task is the interval constructed between the task's minimum duration and maximum duration

$$Dur_i = [min_i, max_i]$$

IPPS ANALYSIS

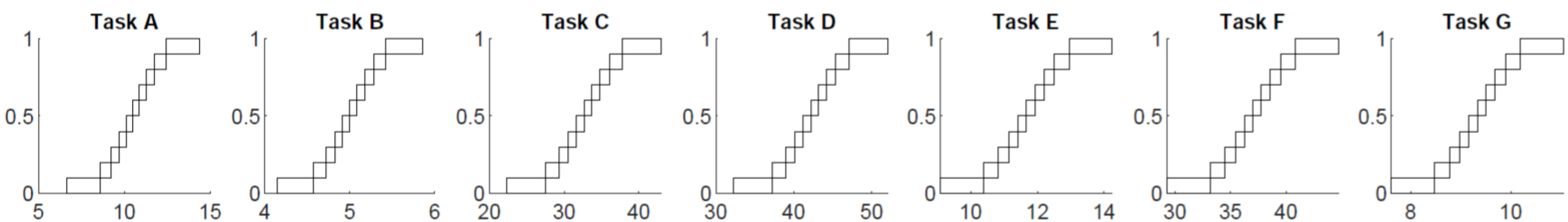
- For each task, no variation is considered in either the mean or standard deviation, $\overline{F}(x)$ coincides with $\underline{F}(x)$
- The imprecise probability structure of each activity is discretized into 10 P-box intervals with equal probability mass.
- A combinatorial analysis (10^7 combinations) is performed (seven-node network with each node with ten P-box intervals).
- As the normal CDF function is continuous, the tails are truncated at

$$P = 0.005, P = 0.995$$

- This establishes finite bounds for the lower-most and upper-most P-box intervals.

IPPS ANALYSIS

Seven P-box interval envelopes for the imprecise probability durations for each task:



ANALYTICAL RESULTS

Task A

Approach	Duration	Early Start	Early Finish	Free Float	Total Float	Late Start	Late Finish
CPM	10	0	10	0	5	5	15
PERT	10.50	0.00	10.50	0.00	4.00	4.00	14.50
IPPS*	[10.50, 10.50]	[0.00, 0.00]	[10.50, 10.50]	[0.00, 0.00]	[3.67, 4.33]	[3.67, 4.33]	[13.83, 15.17]
Interval	[7, 16]	[0, 0]	[7, 16]	[0, 0]	[0, 6]	[0, 6]	[7, 22]

* The results displayed for the IPPS is the interval corresponding to a probability P=0.5

ANALYTICAL RESULTS

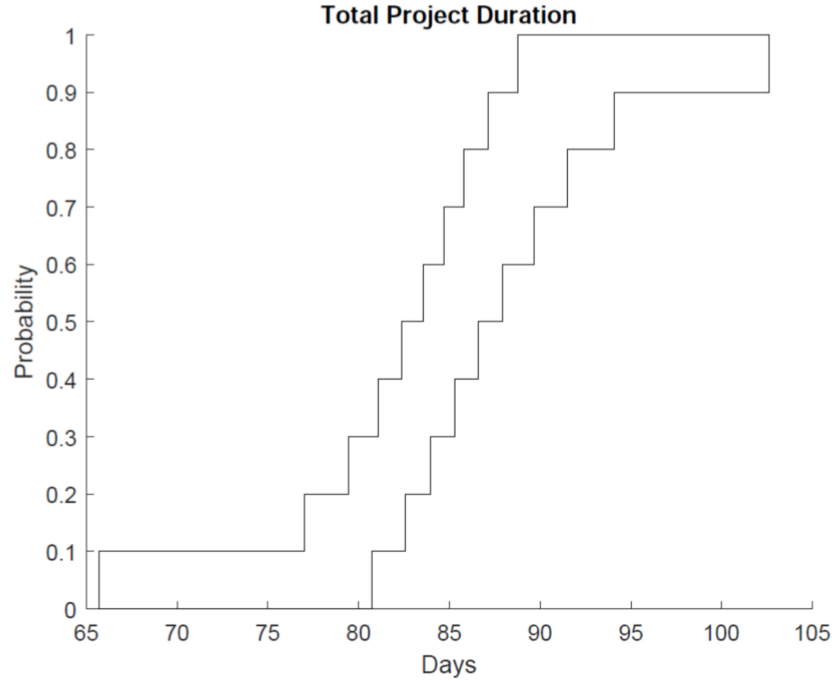
Task F

Approach	Duration	Early Start	Early Finish	Free Float	Total Float	Late Start	Late Finish
CPM	36	45	81	0	0	45	81
PERT	37.00	47.17	84.17	0.00	0.00	47.19	84.17
IPPS*	[37.00, 37.00]	[47.19, 48.75]	[83.54, 86.60]	[0.00, 0.00]	[0.00, 0.00]	[47.17, 48.75]	[83.54, 86.60]
Interval	[30, 48]	[39, 66]	[69, 114]	[0, 0]	[0, 0]	[39, 66]	[69, 114]

* The results displayed for the IPPS is the interval corresponding to a probability P=0.5

ANALYTICAL RESULTS

P-box output for **project duration** calculated as the Early Finish of task F



OBSERVATIONS

- With no assumption on the distribution of the uncertainty, there is sufficient uncertainty in the network tasks to have multiple possible critical paths.
- The interval results enclose both IPPS results.
- As well, the IPPS results enclose the deterministic PERT results.
- This outcome is expected as imprecise probabilities make fewer presumptions than deterministic values, and intervals make no presumptions within their bounds at all.

CONCLUSIONS

- A new formulation for construction project scheduling for a network with temporal uncertainty based on the concepts of imprecise probability (IPPS) is introduced.
- IPPS can obtain the P-box structures for the early start, early finish, free float, total float, late start, and late finish for each activity as well as the critical path and the overall project duration.
- The results obtained from IPPS have a higher level of confidence and robustness because of the objective determination of uncertainties in the parameter distributions.
- The IPPS capability to more robustly enumerate uncertainties makes it attractive to introduce imprecise probability concepts in the field of construction project scheduling and management.

QUESTIONS