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"EXPLICIT SENSITIVITIES OF THE STOCHASTIC RESPONSE OF STRUCTURAL SYSTEMS UNDER SPECTRUM COMPATIBLE FULLY NON-STATIONARY SEISMIC EXCITATIONS"

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Introduction 1/3

- In the framework of the **design** and of the **reliability assessment** of structures, among the static and dynamic loads that have to be considered, certainly the **most important** one is the seismic load.

- The analysis of recorded accelerograms after earthquakes evidence that different earthquakes produce ground motions with different characteristics (intensity, duration, dominant periods and frequency content).

- Time-histories can be considered as sample of a zero mean Gaussian non-stationary processes in both amplitude and frequency content: the so-called **fully non-stationary processes.**

- It is fully characterized by the so-called *evolutionary power spectral density (EPSD) function*, which, for earthquake-resistant structures, should be **compatible** with the **target spectrum** given by the codes.

Introduction

- During the analysis of structural systems, the reference structural parameters could be modified for design reasons (i.e. this is very frequent in the optimization procedures for the design of devices).

- The sensitivity analysis (*evaluation of partial derivatives of a performance measure with respect to system parameters*) is a suitable vehicle to evaluate the variation of the structural systems under the influence of changes of parameter values (Arora and Haug, 1979).

- Since the ground motion acceleration is a non-stationary processes, the sensitivity of the *response evolutionary power spectral density* (EPSD) of structures subjected to non-stationary stochastic processes is an essential information and, consequently, it plays a fundamental role in structural design.

Introduction

Aim: *to propose a novel method for the evaluation of the sensitivities of stochastic response characteristics of structural systems with damping devices subjected to fully non-stationary spectrum compatible excitations*

Since such structural systems are *non-classically damped*, the main steps of the proposed approach are:

1. to write governing motion equations in state-variables
2. to evaluate *the time-frequency varying response vector functions* ***TFR*** in explicit form (Alderucci and Muscolino, 2018);
3. to determine **closed form solutions** of first-order derivatives of the *TFR* as well as of the one-sided *EPSD* of the structural response, with respect to device parameters

Equation of the motion 1/2

- Equation of motion of a quiescent MDOF structural system:

$$\mathbf{M}\ddot{\mathbf{U}}(\boldsymbol{\alpha},t)+\mathbf{C}(\boldsymbol{\alpha}_C)\dot{\mathbf{U}}(\boldsymbol{\alpha},t)+\mathbf{K}(\boldsymbol{\alpha}_K)\mathbf{U}(\boldsymbol{\alpha},t)=-\mathbf{M}\boldsymbol{\tau}\ddot{U}_{\mathrm{g}}(t); \qquad \mathbf{U}(t_0,\boldsymbol{\alpha})=\mathbf{0}$$

$$\boldsymbol{\alpha}^T=\begin{bmatrix}\boldsymbol{\alpha}_c^T & \boldsymbol{\alpha}_k^T\end{bmatrix}$$

$$\boldsymbol{\alpha}=\boldsymbol{\alpha}_0+\Delta\boldsymbol{\alpha}$$

$\Delta\boldsymbol{\alpha}$ → Small parameters variation

$\boldsymbol{\alpha}_0$ → Nominal values of design parameters

$$\mathbf{K}(\boldsymbol{\alpha}_K)=\mathbf{K}_{\mathrm{S}}+\mathbf{K}_{\mathrm{D}}(\boldsymbol{\alpha}_K)$$
$$\mathbf{C}(\boldsymbol{\alpha}_C)=\mathbf{C}_{\mathrm{S}}+\mathbf{C}_{\mathrm{D}}(\boldsymbol{\alpha}_C)$$

- Equation of motion in state variables:

$$\boxed{\dot{\mathbf{Z}}(\boldsymbol{\alpha},t)=\mathbf{D}(\boldsymbol{\alpha})\mathbf{Z}(\boldsymbol{\alpha},t)+\mathbf{w}\ddot{U}_{\mathrm{g}}(t); \quad \mathbf{Z}(\boldsymbol{\alpha},t_0)=\mathbf{0}}$$

$$\mathbf{Z}(\boldsymbol{\alpha},t)=\begin{bmatrix}\mathbf{U}(\boldsymbol{\alpha},t)\\ \dot{\mathbf{U}}(\boldsymbol{\alpha},t)\end{bmatrix}; \quad \mathbf{D}(\boldsymbol{\alpha})=\begin{bmatrix}\mathbf{O}_{n,n} & \mathbf{I}_n\\ -\mathbf{M}^{-1}\mathbf{K}(\boldsymbol{\alpha}_K) & -\mathbf{M}^{-1}\mathbf{C}(\boldsymbol{\alpha}_C)\end{bmatrix}; \quad \mathbf{w}=\begin{bmatrix}\mathbf{O}_{n,1}\\ -\boldsymbol{\tau}\end{bmatrix}; \quad \mathbf{A}(\boldsymbol{\alpha}_C)=\begin{bmatrix}\mathbf{C}(\boldsymbol{\alpha}_C) & \mathbf{M}\\ \mathbf{M} & \mathbf{O}_{n,n}\end{bmatrix}.$$

- It is possible to perform a *complex modal analysis*

$$\mathbf{D}^{-1}(\boldsymbol{\alpha})\boldsymbol{\Psi}(\boldsymbol{\alpha}) = \boldsymbol{\Psi}(\boldsymbol{\alpha})\boldsymbol{\Lambda}^{-1}(\boldsymbol{\alpha})$$

Eigenvalues

Eigenvectors

$$\boldsymbol{\Psi}^T(\boldsymbol{\alpha})\mathbf{A}(\boldsymbol{\alpha}_C)\boldsymbol{\Psi}(\boldsymbol{\alpha}) = \mathbf{I}_{2m}. \qquad \mathbf{A}(\boldsymbol{\alpha}_C) = \begin{bmatrix} \mathbf{C}(\boldsymbol{\alpha}_C) & \mathbf{M} \\ \mathbf{M} & \mathbf{O}_{n,n} \end{bmatrix}.$$

- The following coordinate transformation is introduced:

$$\mathbf{Z}(\boldsymbol{\alpha},t) = \boldsymbol{\Psi}(\boldsymbol{\alpha})\mathbf{X}(\boldsymbol{\alpha},t)$$

Solution in the nodal space

Solution in the complex modal subspace

Deterministic sensitivity function 1/2

- Sensitivity analysis consist in the evaluation of the change in the system response due to system parameter variations in the neighbourhood of prefixed values, called “nominal parameter”.

- The sensitivity vector of the structural response in state variables with respect to the i-th parameter α_i of the $\boldsymbol{\alpha}$ vector

$$\boxed{\mathbf{s}_{\mathbf{Z},i}(t,\boldsymbol{\alpha}_0)=\left.\frac{\partial \mathbf{Z}(t,\boldsymbol{\alpha})}{\partial \alpha_i}\right|_{\boldsymbol{\alpha}=\boldsymbol{\alpha}_0}}$$

can be obtained by differentiating the equation of motion with respect to $\boldsymbol{\alpha}$, setting $\boldsymbol{\alpha}=\boldsymbol{\alpha}_0$

$$\dot{\mathbf{s}}_{\mathbf{Z},i}(\boldsymbol{\alpha}_0,t)=\mathbf{D}(\boldsymbol{\alpha}_0)\mathbf{s}_{\mathbf{Z},i}(\boldsymbol{\alpha}_0,t)+\mathbf{D}_i'(\boldsymbol{\alpha}_0)\mathbf{Z}(\boldsymbol{\alpha}_0,t); \quad \mathbf{s}_{\mathbf{Z},i}(\boldsymbol{\alpha}_0,t_0)=\mathbf{0}$$

$$\boxed{\mathbf{s}_{\mathbf{Z},i}(t,\boldsymbol{\alpha}_0)=\int_{t_0}^{t}\boldsymbol{\Theta}(t-\tau,\boldsymbol{\alpha}_0)\mathbf{D}_i'(\boldsymbol{\alpha}_0)\boldsymbol{\Psi}(\boldsymbol{\alpha}_0)\mathbf{X}(t,\boldsymbol{\alpha}_0)\,\mathrm{d}\tau}$$

$$\boldsymbol{\Theta}(t,\boldsymbol{\alpha})=\exp\left[t\mathbf{D}(\boldsymbol{\alpha})\right]=\boldsymbol{\Psi}(\boldsymbol{\alpha})\exp\left[t\boldsymbol{\Lambda}(\boldsymbol{\alpha})\right]\boldsymbol{\Psi}^{T}(\boldsymbol{\alpha})\mathbf{A}(\boldsymbol{\alpha}_C)$$

Deterministic sensitivity function 2/2

- Alternatively, the state-variable sensitivity vector with respect to the i-th parameter, can be evaluated as

$$\mathbf{s}_{\mathbf{Z},i}(t,\boldsymbol{\alpha}_0) = \boldsymbol{\Psi}(\boldsymbol{\alpha}_0)\mathbf{Y}_i(t,\boldsymbol{\alpha}_0)$$

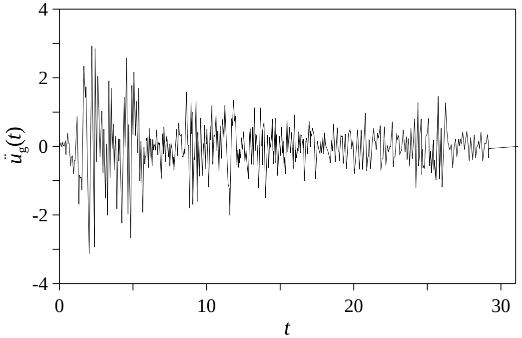
sensitivity vector of the response into the complex modal subspace

$$\boxed{\mathbf{Y}_i(t,\boldsymbol{\alpha}_0) = \int_{t_0}^{t} \exp\left[(\tau-\rho)\boldsymbol{\Lambda}(\boldsymbol{\alpha}_0)\right]\mathbf{B}_i(\boldsymbol{\alpha}_0)\mathbf{X}(\tau,\boldsymbol{\alpha}_0)\,\mathrm{d}\tau}$$

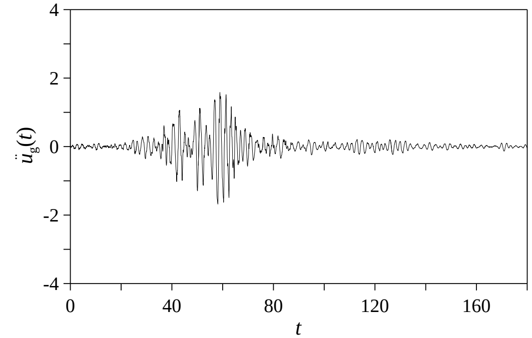
$$\mathbf{B}_i(\boldsymbol{\alpha}_0) = \boldsymbol{\Psi}^T(\boldsymbol{\alpha}_0)\mathbf{A}(\boldsymbol{\alpha}_{C,0})\mathbf{D}'_i(\boldsymbol{\alpha}_0)\boldsymbol{\Psi}(\boldsymbol{\alpha}_0); \quad \mathbf{v}(\boldsymbol{\alpha}_0) = \boldsymbol{\Psi}^T(\boldsymbol{\alpha}_0)\mathbf{A}(\boldsymbol{\alpha}_{C,0})\mathbf{w}$$

- Notice that for deterministic excitation the state-variable sensitivity vectors with respect to the i-th uncertain parameter, can be easily evaluated by step-by-step procedures (Cacciola et al, 2005).

Seismic accelerations as fully non-stationary random processes 1/2



El Centro NS recorded earthquake 1940.



Mexico City N90W recorded earthquake 1985.

- The ground motion acceleration, $\ddot{U}_{\mathrm{g}}(t)$, is herein assumed as a zero-mean Gaussian **fully non-stationary random process**, defined by the Priestley spectral representation (Priestley, 1965; 1967).

- The zero-mean Gaussian fully non-stationary random process is a complex process.

$$\boxed{\ddot{U}_{\mathrm{g}}(t) = \sqrt{2}\int_0^\infty \exp(\mathrm{i}\omega t)\, a(\omega,t)\, \mathrm{d}N(\omega)}$$

$$a(\omega,t) \equiv a^*(-\omega,t) \qquad \mathrm{E}\left\langle \mathrm{d}N(\omega_1)\,\mathrm{d}N^*(\omega_2)\right\rangle = \delta(\omega_1-\omega_2)\,G_0(\omega_1)\,\mathrm{d}\omega_1\,\mathrm{d}\omega_2$$

One sided PSD

Seismic accelerations as fully non-stationary random processes 2/2

- The complex process $\ddot{U}_g(t)$ can be completely defined in the time domain by the knowledge of its complex autocorrelation function.

$$R_{\ddot{U}_g\ddot{U}_g}(t_1,t_2) \equiv \mathrm{E}\left\langle \ddot{U}_g(t_1)\ddot{U}_g(t_2)\right\rangle = \int_0^\infty \exp\left[\mathrm{i}\omega\left(t_1-t_2\right)\right]\underbrace{a(\omega,t_1)\,a^*(\omega,t_2)G_0(\omega)}_{\boxed{G_{\ddot{U}_g\ddot{U}_g}(\omega,t)=\left|a(\omega,t)\right|^2 G_0(\omega)}}\mathrm{d}\omega$$

$\boxed{\ddot{U}_g(t)}$

pre-envelope process (Di Paola ,1985).

one sided EPSD function

- The seismic excitations, is herein modeled as **fully non-stationary spectrum compatible processes**.
- It is well known that for fully non-stationary random model the *spectrum compatible EPSD* function cannot be defined univocally (Cacciola 2010). Here the iterative procedure recently proposed by (Alderucci et al, 2019) is adopted.

Closed form solution for the sensitivity time-frequency varying response vector function 1/3

- The *time-frequency varying response* (*TFR*) vector function of the response plays a central role in the evaluation of the statistics of the response for both classically and non-classically damped structural systems subjected to fully non-stationary stochastic input

$$\mathbf{Z}(\omega,t,\boldsymbol{\alpha}) = \boldsymbol{\Psi}(\boldsymbol{\alpha})\mathbf{X}(\omega,t,\boldsymbol{\alpha})$$

MTFR

(Muscolino and Alderucci 2015, 2018)

$$\boldsymbol{\alpha} = \boldsymbol{\alpha}_0 + \Delta\boldsymbol{\alpha}$$

unknown r-order parameter vector

- For the Spanos and Solomos (1983) model

$$a(\omega,t) = \varepsilon(\omega)\,(t-t_0)\exp\left[-\alpha_a(\omega)(t-t_0)\right]\mathbb{U}(t-t_0);$$

$$\boxed{\mathbf{X}(\omega,t,\boldsymbol{\alpha}) = -\varepsilon(\omega)\left\{\exp\left(-\beta(\omega)\,t\right)\left[\boldsymbol{\Gamma}^2(\omega,\boldsymbol{\alpha}) + t\,\boldsymbol{\Gamma}(\omega,\boldsymbol{\alpha})\right] - \exp\left[t\,\boldsymbol{\Lambda}(\boldsymbol{\alpha})\right]\boldsymbol{\Gamma}^2(\omega,\boldsymbol{\alpha})\right\}\mathbf{v}(\boldsymbol{\alpha})\,\mathbb{U}(t)}$$

$$\beta(\omega) = \alpha_a(\omega) - \mathrm{i}\omega \qquad \boldsymbol{\Gamma}(\omega,\boldsymbol{\alpha}) = \left[\boldsymbol{\Lambda}(\boldsymbol{\alpha}) + \beta(\omega)\mathbf{I}_{2m}\right]^{-1}$$

Closed form solution for the sensitivity time-frequency varying response vector function 2/3

- The sensitivity of the TFR vector function

$$\boxed{\mathbf{s}_{\mathbf{Z},i}\left(\omega,t,\boldsymbol{\alpha}_0\right)=\boldsymbol{\Psi}\left(\boldsymbol{\alpha}_0\right)\mathbf{Y}_i\left(\omega,t,\boldsymbol{\alpha}_0\right)} \quad \text{MSTFR}$$

- $\mathbf{Y}_i(\boldsymbol{\alpha}_0,\omega,t)$ is the *modal sensitivity TFR* vector function with respect to the parameter α_i . It can be evaluated as solution of the following differential equation with zero start conditions at initial time:

$$\boxed{\dot{\mathbf{Y}}_i(\omega,t,\boldsymbol{\alpha}_0)=\boldsymbol{\Lambda}(\boldsymbol{\alpha}_0)\mathbf{Y}_i(\omega,t,\boldsymbol{\alpha}_0)+\mathbf{B}_i\left(\boldsymbol{\alpha}_0\right)\mathbf{X}(\omega,\tau,\boldsymbol{\alpha}_0)\mathbb{U}\left(t-t_0\right); \quad \mathbf{Y}_i(\omega,0,\boldsymbol{\alpha}_0)=\mathbf{0}.}$$

- The equations of the MTFR vector (Alderucci, Genovese and Muscolino 2019) are rewritten as:

$$\mathbf{X}\left(\omega,t,\boldsymbol{\alpha}_0\right)=\mathbf{X}_1\left(\omega,t,\boldsymbol{\alpha}_0\right)+\mathbf{X}_2\left(\omega,t,\boldsymbol{\alpha}_0\right)$$

$$\boxed{\begin{aligned}&\mathbf{X}_1\left(\omega,t,\boldsymbol{\alpha}_0\right)=-\varepsilon(\omega)\exp\left(-\beta(\omega)\,t\right)\left[\boldsymbol{\Gamma}_0^2(\omega)+t\,\boldsymbol{\Gamma}_0(\omega)\right]\mathbf{v}_0\,\mathcal{U}(t);\\&\mathbf{X}_2\left(\omega,t,\boldsymbol{\alpha}_0\right)=\varepsilon(\omega)\exp\left(t\,\boldsymbol{\Lambda}_0\right)\boldsymbol{\Gamma}_0^2(\omega)\mathbf{v}_0\,\mathcal{U}(t).\end{aligned}}$$

Closed form solution for the sensitivity time-frequency varying response vector function 3/3

- It follows that the *MSTFR vector function* can be evaluate in closed form solution as:

$$\boxed{\begin{aligned}&\mathbf{Y}_i(\omega,t,\boldsymbol{\alpha}_0)=\mathbf{Y}_{i,1}(\omega,t,\boldsymbol{\alpha}_0)+\mathbf{Y}_{i,2}(\omega,t,\boldsymbol{\alpha}_0)=\\&=\left\{\mathbf{Y}_{i,1,\mathrm{p}}(\omega,t,\boldsymbol{\alpha}_0)+\mathbf{Y}_{i,2,\mathrm{p}}(\omega,t,\boldsymbol{\alpha}_0)-\exp(t\,\boldsymbol{\Lambda}_0)\left[\mathbf{Y}_{i,1,\mathrm{p}}(\omega,0,\boldsymbol{\alpha}_0)+\mathbf{Y}_{i,2,\mathrm{p}}(\omega,0,\boldsymbol{\alpha}_0)\right]\right\};\quad t>0\end{aligned}}$$

- The particular solution vector are determined as follows:

$$\boxed{\begin{aligned}&\mathbf{Y}_{i,1,\mathrm{p}}(\omega,t,\boldsymbol{\alpha}_0)=\varepsilon(\omega)\exp(-\beta(\omega)\,t)\;\boldsymbol{\Gamma}(\omega,\boldsymbol{\alpha}_0)\left[\boldsymbol{\Gamma}(\omega,\boldsymbol{\alpha}_0)\mathbf{B}_i(\boldsymbol{\alpha}_0)+\mathbf{B}_i(\boldsymbol{\alpha}_0)\boldsymbol{\Gamma}(\omega,\boldsymbol{\alpha}_0)+t\,\mathbf{B}_i(\boldsymbol{\alpha}_0)\right]\boldsymbol{\Gamma}(\omega,\boldsymbol{\alpha}_0)\mathbf{v}(\boldsymbol{\alpha}_0);\\&\mathbf{Y}_{i,2,\mathrm{p}}(\omega,t,\boldsymbol{\alpha}_0)=\varepsilon(\omega)\mathbf{P}_i(t,\boldsymbol{\alpha}_0)\exp\left[t\,\boldsymbol{\Lambda}(\boldsymbol{\alpha}_0)\right]\boldsymbol{\Gamma}^2(\omega,\boldsymbol{\alpha}_0)\mathbf{v}(\boldsymbol{\alpha}_0);\end{aligned}}$$

$$P_{i,jj}(t,\boldsymbol{\alpha}_0)=t\;B_{i,jj}(\boldsymbol{\alpha}_0);\quad P_{i,jk}(t,\boldsymbol{\alpha}_0)=\frac{B_{i,jk}(\boldsymbol{\alpha}_0)}{\lambda_k-\lambda_j},\; j\neq k$$

Closed form solutions for the sensitivities of the EPSD response matrix function 1/3

- The stochastic response is a zero-mean fully non-stationary stochastic vector process too, whose *one-sided EPSD* matrix function can be evaluated as follows (Alderucci, Genovese and Muscolino 2019):

$$\boxed{\mathbf{G}_{\mathbf{ZZ}}(\omega,t,\boldsymbol{\alpha}) = G_0(\omega)\,\mathbf{Z}^*(\omega,t,\boldsymbol{\alpha})\mathbf{Z}^T(\omega,t,\boldsymbol{\alpha}) = G_0(\omega)\,\boldsymbol{\Psi}^*(\boldsymbol{\alpha})\mathbf{X}^*(\omega,t,\boldsymbol{\alpha})\mathbf{X}^T(\omega,t,\boldsymbol{\alpha})\boldsymbol{\Psi}^T(\boldsymbol{\alpha})}$$

$G_0(\omega)$ one-sided *PSD* function of the "embedded" stationary counterpart of the input process

$\mathbf{Z}(\omega,t,\boldsymbol{\alpha})$ TFR vector responses in the nodal space

$\mathbf{X}(\omega,t,\boldsymbol{\alpha})$ TFR vector responses in the modal complex space

$$\boxed{\boldsymbol{\Sigma}_{\mathbf{ZZ}}(t,\boldsymbol{\alpha}) = \boldsymbol{\Psi}^*(\boldsymbol{\alpha})\left[\int_0^\infty G_0(\omega)\,\mathbf{X}^*(\omega,t,\boldsymbol{\alpha})\,\mathbf{X}^T(\omega,t,\boldsymbol{\alpha})\,\mathrm{d}\,\omega\right]\boldsymbol{\Psi}^T(\boldsymbol{\alpha})}$$

- This matrix is called pre-envelope covariance (PEC) matrix function, in nodal space; it is a $2n \times 2n$ Hermitian matrix, whose real part coincides with the classical covariance matrix (Di Paola, 1985).

Closed form solutions for the sensitivities of the EPSD response matrix function 2/3

- By differentiating the *PEC* matrix, it is possible to evaluate its sensitivity with respect to the *i*-th parameter, in the neighbourhood of nominal parameters, $\boldsymbol{\alpha}=\boldsymbol{\alpha}_0$, as follows:

$$\boldsymbol{\Sigma}_{\mathbf{s}_{\mathbf{Z},i}}\left(t,\boldsymbol{\alpha}_0\right)=\left.\frac{\partial\boldsymbol{\Sigma}_{\mathbf{ZZ}}(t,\boldsymbol{\alpha})}{\partial\alpha_i}\right|_{\boldsymbol{\alpha}=\boldsymbol{\alpha}_0}=\mathrm{E}\left\langle\mathbf{Z}(t,\boldsymbol{\alpha}_0)\mathbf{s}_{\mathbf{Z},i}^{*T}\left(t,\boldsymbol{\alpha}_0\right)\right\rangle+\mathrm{E}\left\langle\mathbf{Z}(t,\boldsymbol{\alpha}_0)\mathbf{s}_{\mathbf{Z},i}^{*T}\left(t,\boldsymbol{\alpha}_0\right)\right\rangle^{*T}$$

$$\boxed{\mathrm{E}\left\langle\mathbf{Z}^*(\boldsymbol{\alpha}_0,t)\mathbf{s}_{\mathbf{Z},i}^{T}\left(\boldsymbol{\alpha}_0,t\right)\right\rangle=\boldsymbol{\Psi}^*(\boldsymbol{\alpha}_0)\left\{\int_0^\infty\mathbf{X}^*(\boldsymbol{\alpha}_0,\omega,t)\mathbf{Y}_i^T(\boldsymbol{\alpha}_0,\omega,t)\,G_0(\omega)\mathrm{d}\omega\right\}\boldsymbol{\Psi}^T\left(\boldsymbol{\alpha}_0\right)}$$

$$\boxed{\mathbf{Y}\left(\boldsymbol{\alpha}_0,\omega,t\right)=\left.\frac{\partial}{\partial\alpha_i}\mathbf{X}(\boldsymbol{\alpha}_0,\omega,t)\right|_{\boldsymbol{\alpha}=\boldsymbol{\alpha}_0}}$$

Closed form solutions for the sensitivities of the EPSD response matrix function 3/3

Sensitivity of PEC Matrix

$$\boxed{\boldsymbol{\Sigma}_{\mathbf{s}_{\mathbf{Z},i}}(t,\boldsymbol{\alpha}_0) = \frac{\partial}{\partial \alpha_i}\boldsymbol{\Sigma}_{\mathbf{ZZ}}(t,\boldsymbol{\alpha})\bigg|_{\boldsymbol{\alpha}=\boldsymbol{\alpha}_0} \equiv \int_0^\infty \mathbf{G}_{\mathbf{s}_{\mathbf{Z},i}}(\omega,t,\boldsymbol{\alpha}_0)\,\mathrm{d}\omega}$$

- whose elements are the sensitivity of first three spectral moments with respect to the parameter α_i

$$\mathbf{G}_{\mathbf{s}_{\mathbf{Z},i}}(\omega,t,\boldsymbol{\alpha}_0) = \frac{\partial \mathbf{G}_{\mathbf{ZZ}}(\omega,t,\boldsymbol{\alpha})}{\partial \alpha_i}\bigg|_{\boldsymbol{\alpha}=\boldsymbol{\alpha}_0}$$

$$= G_0(\omega)\,\boldsymbol{\Psi}^*(\boldsymbol{\alpha}_0)\left[\mathbf{X}^*(\omega,t,\boldsymbol{\alpha}_0)\,\mathbf{Y}_i^T(\omega,t,\boldsymbol{\alpha}_0) + \mathbf{Y}_i^*(\omega,t,\boldsymbol{\alpha}_0)\,\mathbf{X}^T(\omega,t,\boldsymbol{\alpha}_0)\right]\boldsymbol{\Psi}^T(\boldsymbol{\alpha}_0).$$

$\mathbf{G}_{\mathbf{s}_{\mathbf{Z},i}}(\boldsymbol{\alpha}_0,t)$ is the sensitivity of the one-sided *EPSD* function matrix of nodal response

Numerical Application 1/7

- In order to show the effectiveness of the proposed method, 4 **SDOF** systems are analysed.

$$T_{0,1} = 0.1\ s$$
$$T_{0,2} = 0.2\ s$$
$$T_{0,3} = 0.6\ s$$
$$T_{0,4} = 1\ s$$

$$\zeta_0 = 0.02$$

- An external damper device, with damping coefficient c_d and stiffness k_d is connected to each SDOF system.

- For the SDOF system the PEC matrix function, in nodal space, it is a Hermitian matrix, whose real part coincides with the classical covariance matrix.

$$\mathbf{\Sigma_{zz}}(t,\boldsymbol{\alpha}) = \begin{bmatrix} \lambda_{0,u}(t,\boldsymbol{\alpha}) & \mathrm{i}\,\lambda_{1,u}(t,\boldsymbol{\alpha}) \\ -\mathrm{i}\,\lambda_{1,u}^{*T}(t,\boldsymbol{\alpha}) & \lambda_{2,u}(t,\boldsymbol{\alpha}) \end{bmatrix}$$

non-geometric spectral moments (NGSM) of i-th order of stochastic response (Michealov et al, 1999). .

Numerical Application

- The selected structures are subjected to a fully non-stationary spectrum-compatible seismic input

$$G_{FF}(\omega,t) = a^2(\omega,t)\, G_0(\omega)$$

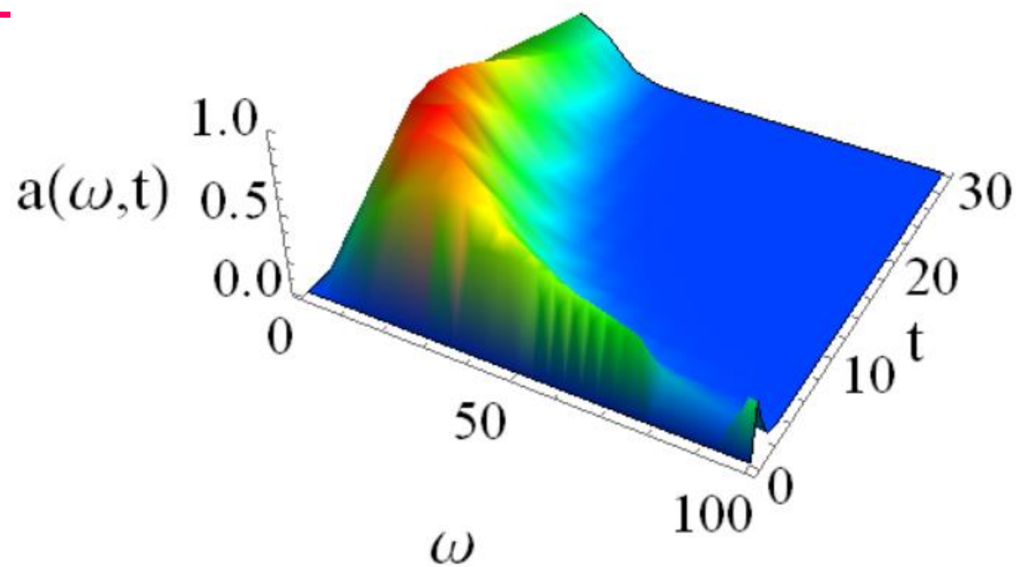
- The Spanos and Solomos (1983) time-modulating-function has been set.

$$\boxed{a(\omega,t) = \varepsilon(\omega)\, t \exp\left(-\alpha(\omega)t\right)\mathcal{U}(t)}$$

$$\varepsilon(\omega) = \frac{\sqrt{2}}{15\pi a_{\max}}\omega$$

$$a_{\max} = 1.34$$

$$\alpha_a(\omega) = \frac{1}{2}\left(0.15 + \frac{\omega^2}{225\pi^2}\right)$$

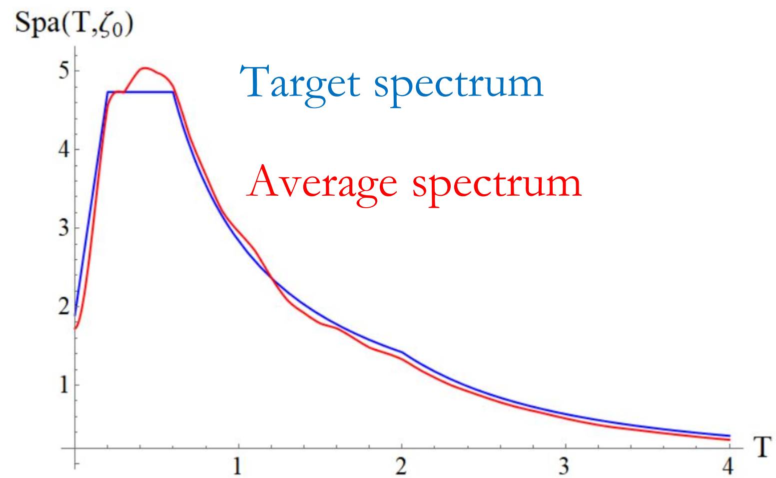


The target spectrum is obtained following the EC8 instructions

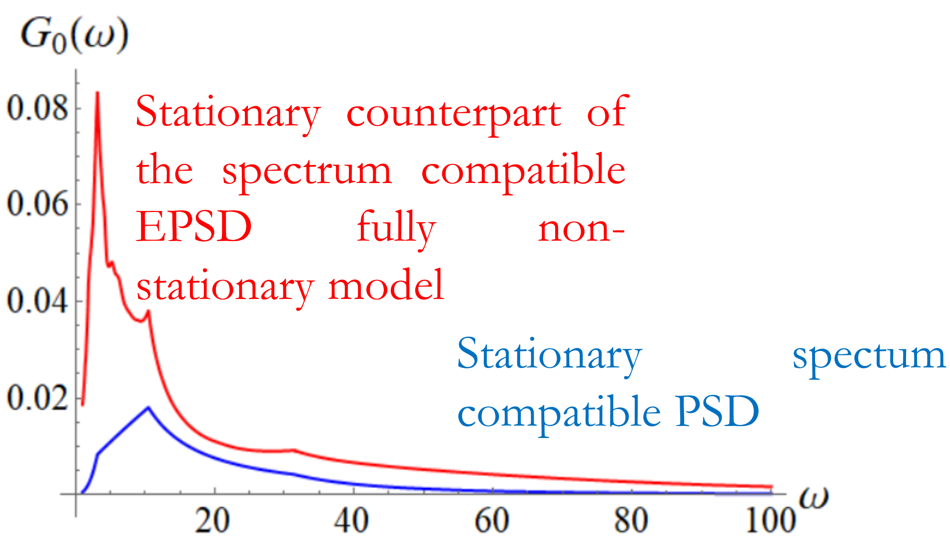
$$T_B = 0.2 \text{ sec} \qquad S = 1.15$$

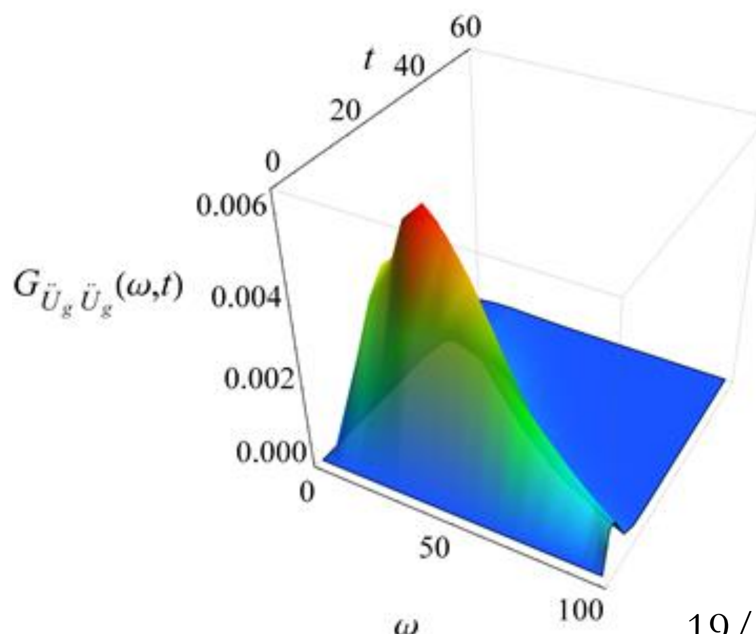
$$T_C = 0.6 \text{ sec} \qquad a_g = 1.647 \text{ m/sec}^2$$

$$T_D = 2.0 \text{ sec}$$



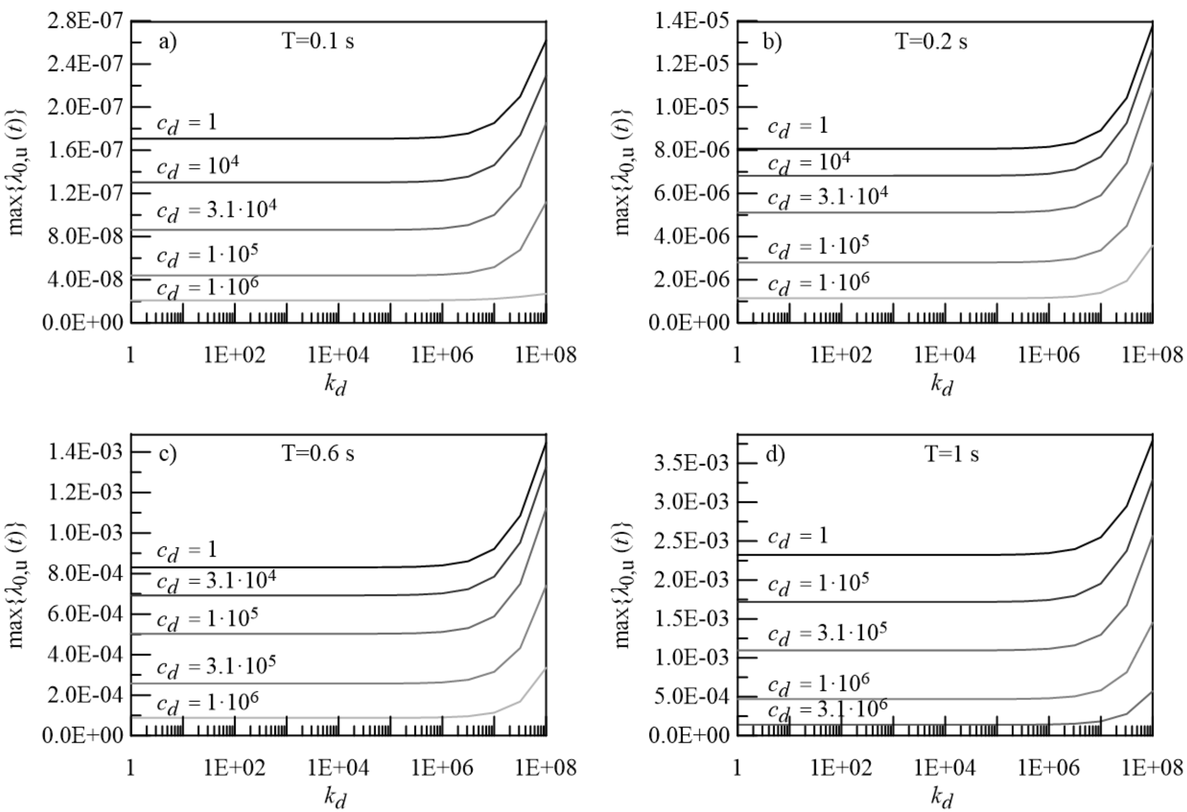
- The spectrum compatibility is obtained through the formulation proposed by Alderucci et al (2019)





Numerical Application

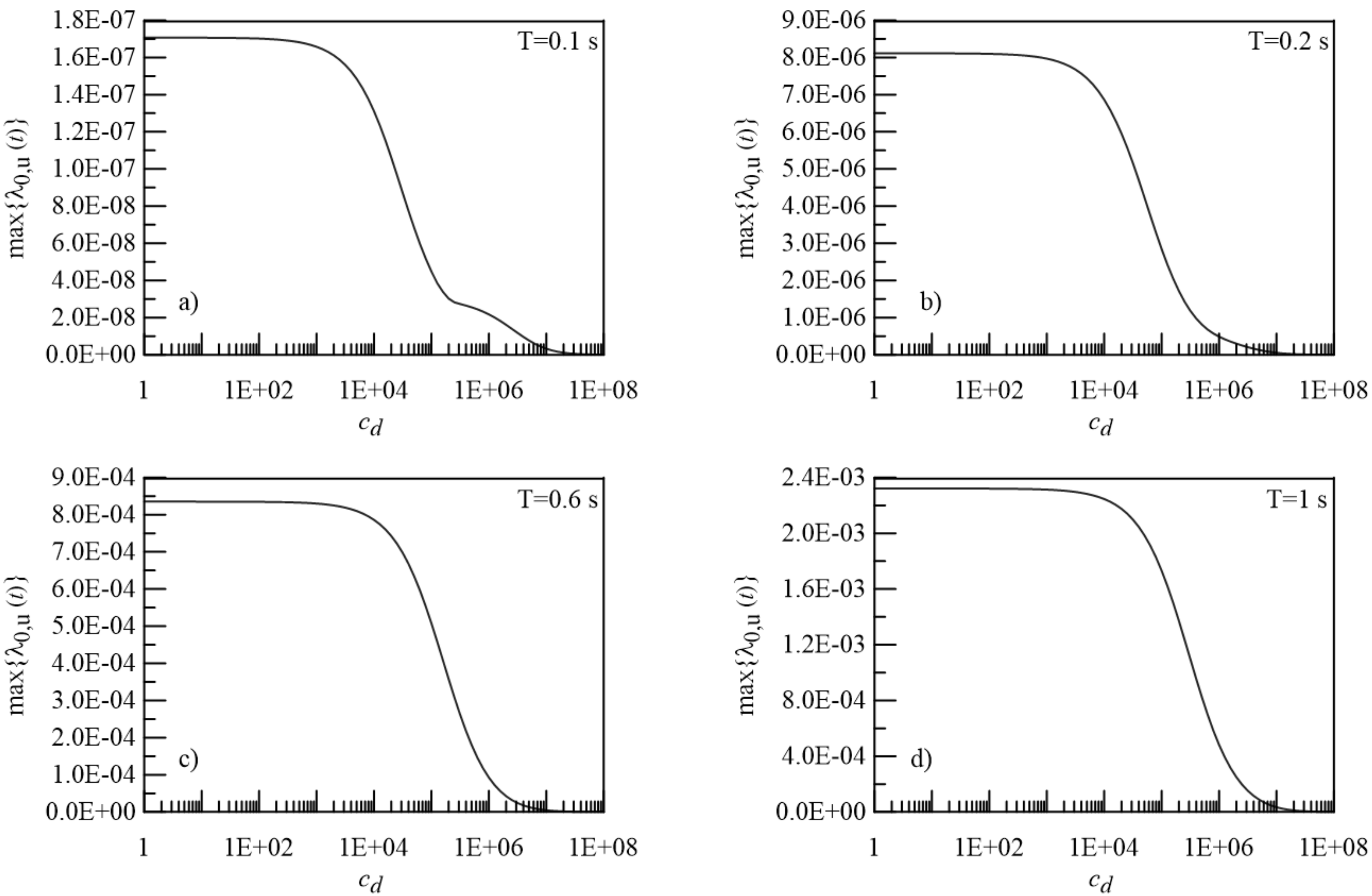
- In order to choose the *best damper stiffness of the device*, a parametric analysis has been conducted to define the optimal stiffness value, varying the parameter in the range $1 \div 10^8$ N/m



The chosen stiffness value of the fluid dampers devices is 30 N/m, since the parameter changes don't affect significantly the response

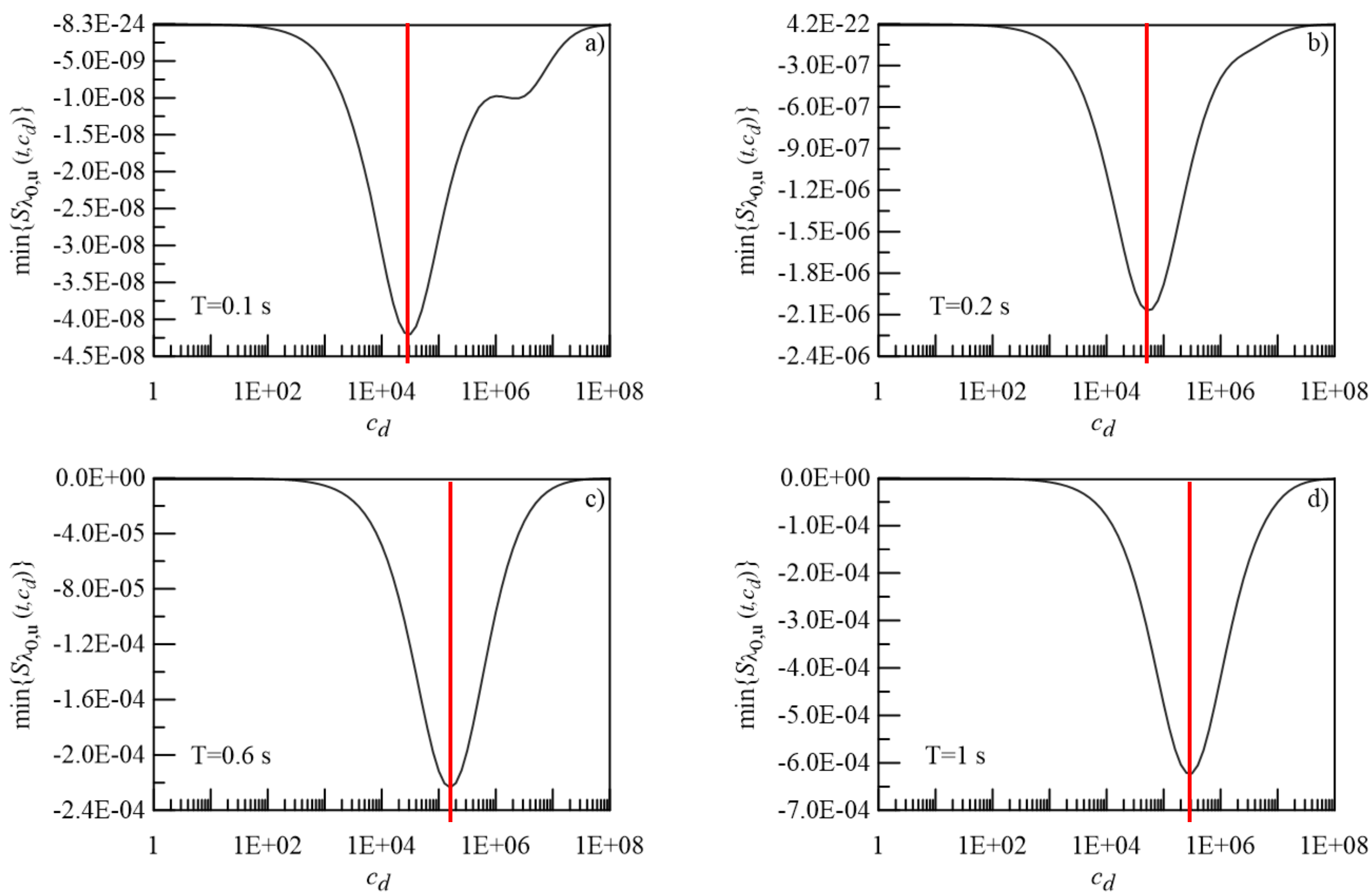
Numerical Application 5/7

- The *maximum values of the variance of the response*, versus the damping coefficient, for the previously chosen optimal value of the stiffness

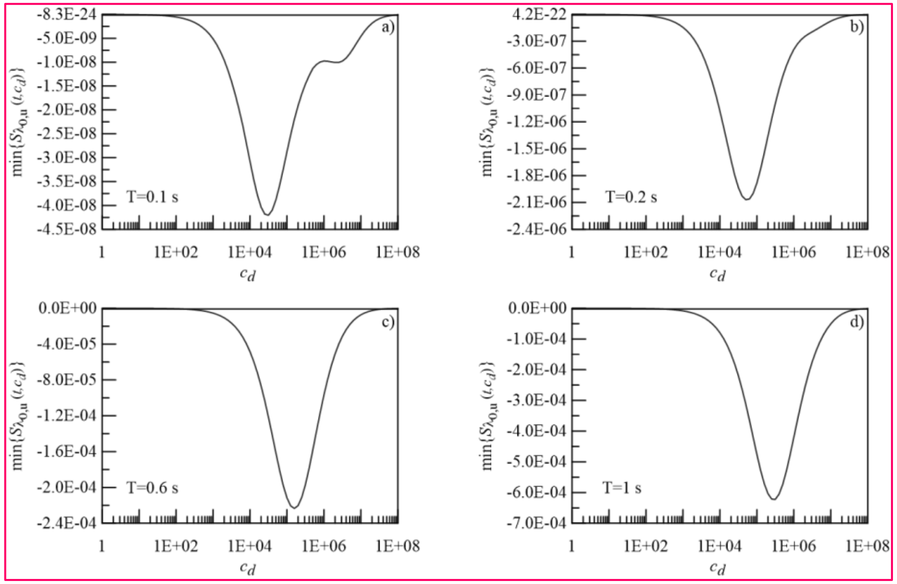


- In order to define the best damping coefficient value, a parametric study has been conducted, analyzing the sensitivity of nodal response

$$\min\left\{S_{\lambda_0}(t,c_d)\right\} = \\ = \min\left\{\left|\partial\lambda_{0,u}(t)/\partial c_d\right|_{k_d=k_{d,0}}\right\}$$



- It is well known that for small variation of a parameter with respect the nominal one, it is possible to predict with good accuracy the variation of the response spectral moment by the knowledge of its sensitivity.



- The **optimal damping coefficients**, are those corresponding to the points at which the minimum values of the sensitivity functions, assume the smallest values.

T [s]	c_d [Ns/m]
0.1	31 623
0.2	50 119
0.6	158 489
1.0	310 000

Concluding Remarks

- The present work aimed to define a *new method* to evaluate *sensitivities of stochastic response characteristics* of structural systems subjected to seismic excitations;

- The ground motion acceleration was herein modelled as fully non-stationary spectrum compatible Gaussian stochastic processes.

- **Closed form solutions** for the first-order derivatives of the TFR as well as of the one-sided evolutionary PSD (EPSD) of the structural response, with respect to damping parameters of devices, are evaluated.

- The numerical application on different SDOF oscillators showed the accuracy and the computational efficiency of the proposed method

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Thanks for the attention

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