9th International Workshop on Reliable Engineering Computing Risk and Uncertainty in Egnineering Computations

Free vibration of functionally graded graphene-platelets reinforced composite plate with interval parameters

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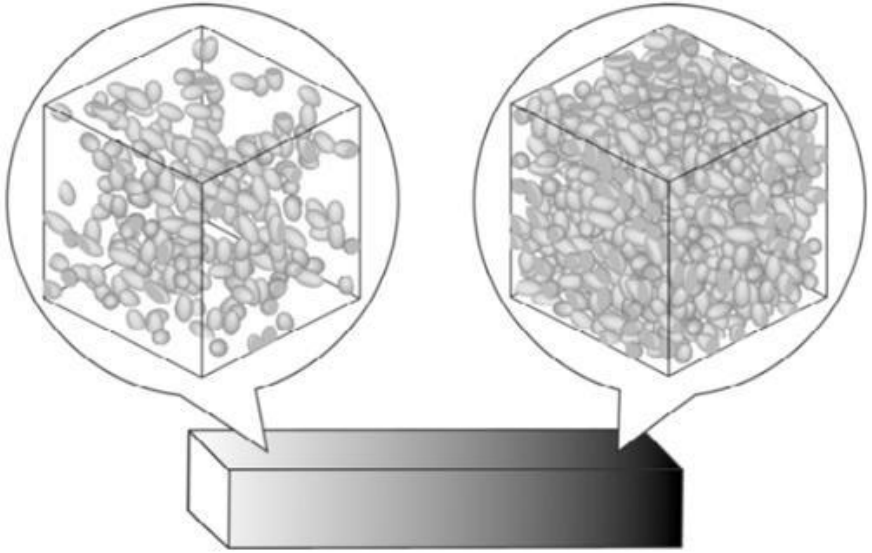
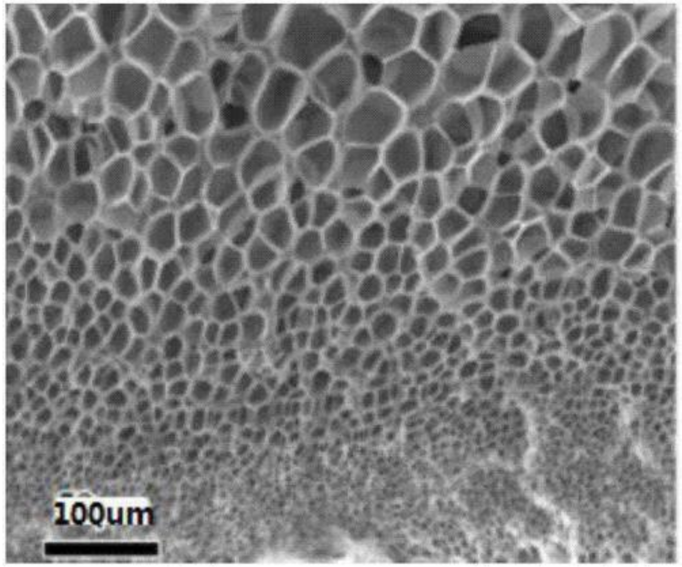
Outline

- Introduction
- Free vibration of FG-GPLRC plate
- Interval eigenvalue analysis
- Results and discussions
- Conclusions

Introduction

- With the development of science and technology, functionally graded composite materials are widely used. Various types of FG materials have been proposed.

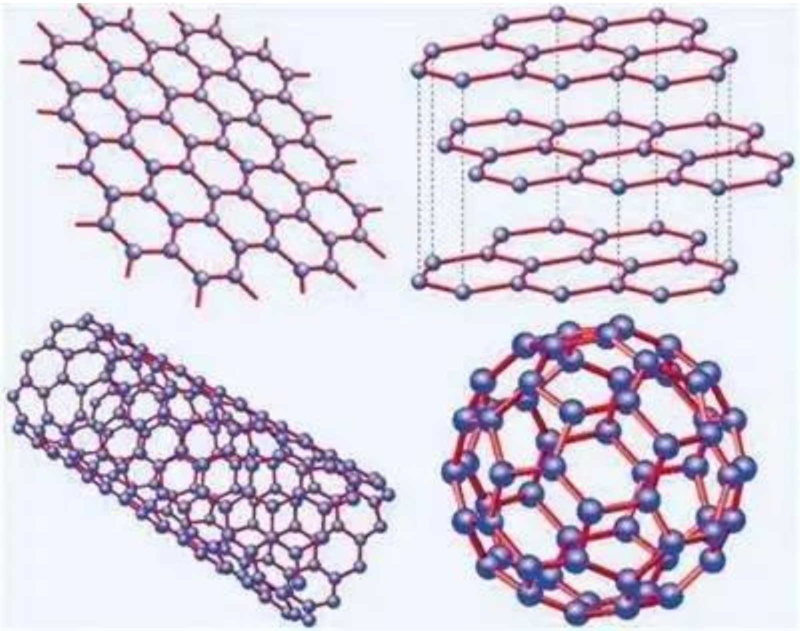


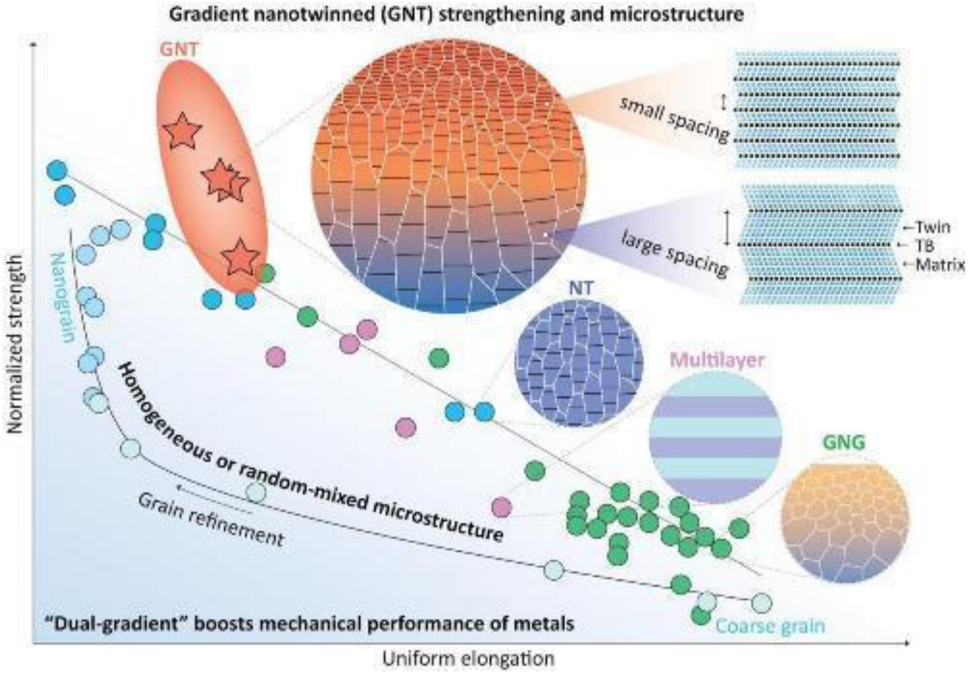


- In FG materials, material composition is tailored to continuously vary over one or more dimension(s) to simultaneously meet different mechanical requirements.

◆ Introduction

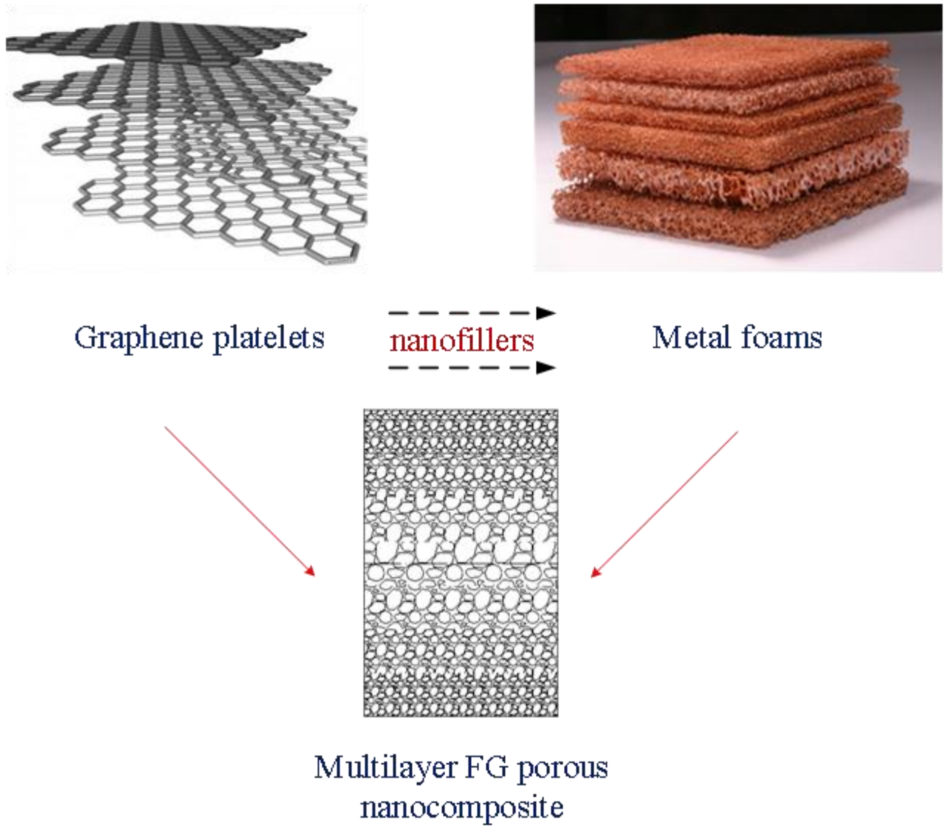
- Owing to extraordinary mechanical, thermal and electrical properties of nano-materials, the nano-FG composites have attracted worldwide attentions in the field of composite materials.

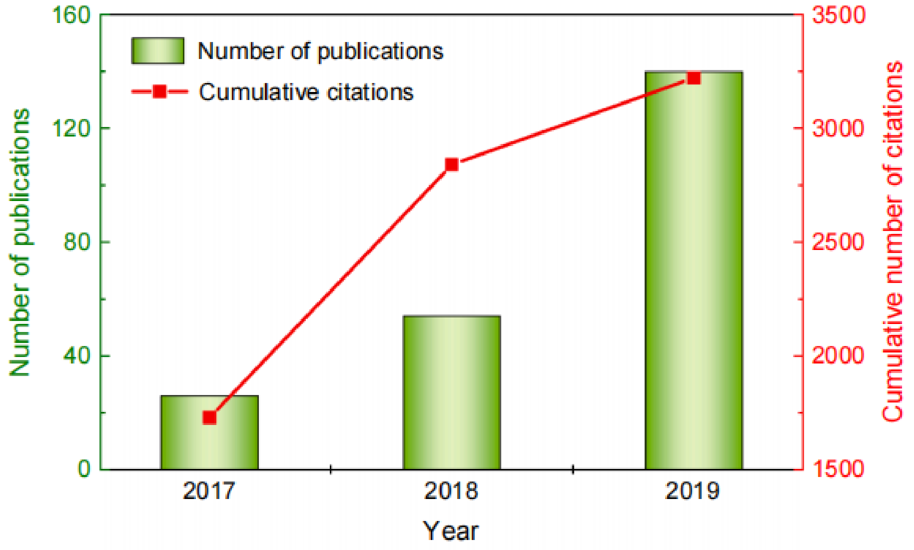


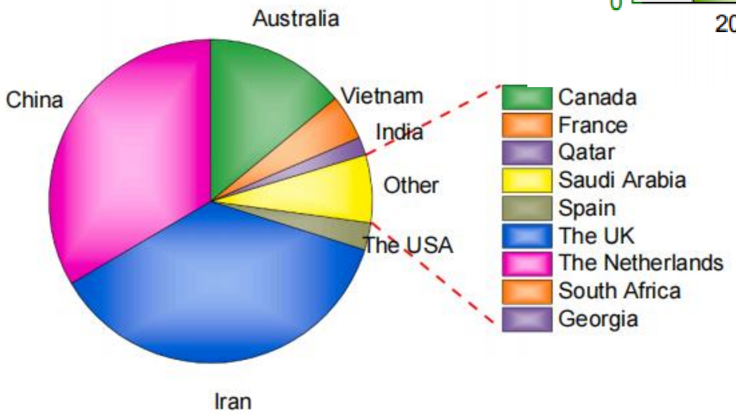


◆ Introduction

- Compared with other carbon-based nanofillers, graphene has extremely high specific surface area, which make them excellent candidates as the reinforcement materials in nanocomposites.
- Recently, a new type of FG graphene-platelets (GPLs) reinforced nanocomposites (FG-GPLRC) have been proposed.
- Publications concerning FG-GPLRC has been growing extensively

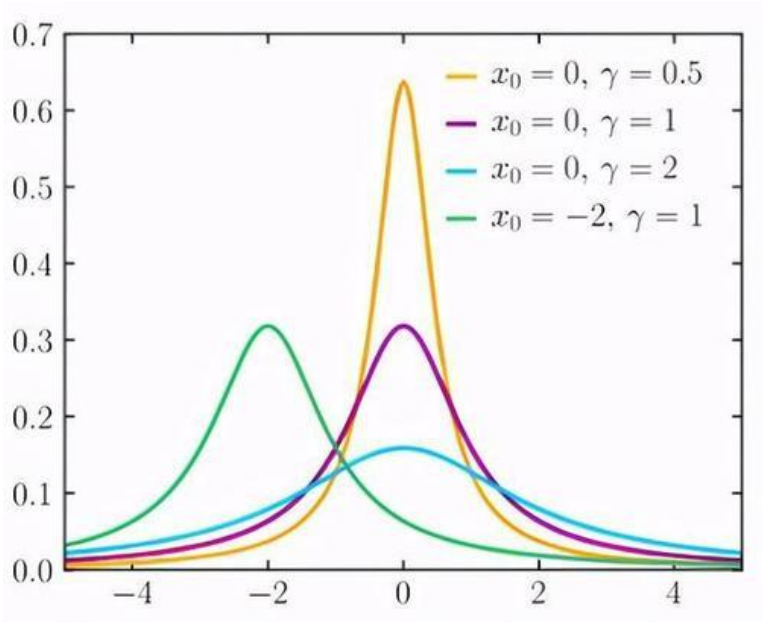


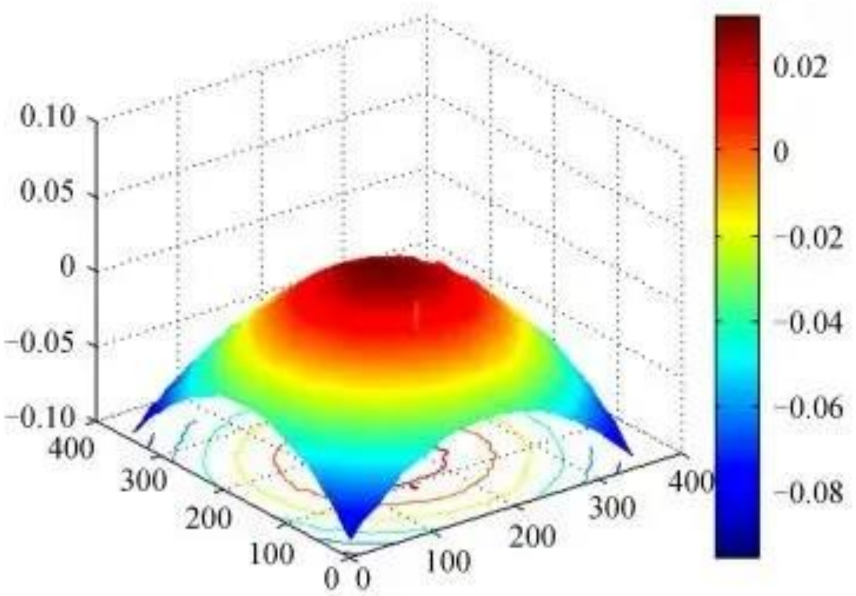




Introduction

- Material properties of composites are inevitably subject to some degree of uncertainty due to complicated manufacturing processes and inherent nonuniform dispersion of constituents.
- Uncertainties in composites are usually modelled as random variables and the uncertainty propagations are analyzed by the probabilistic methods.

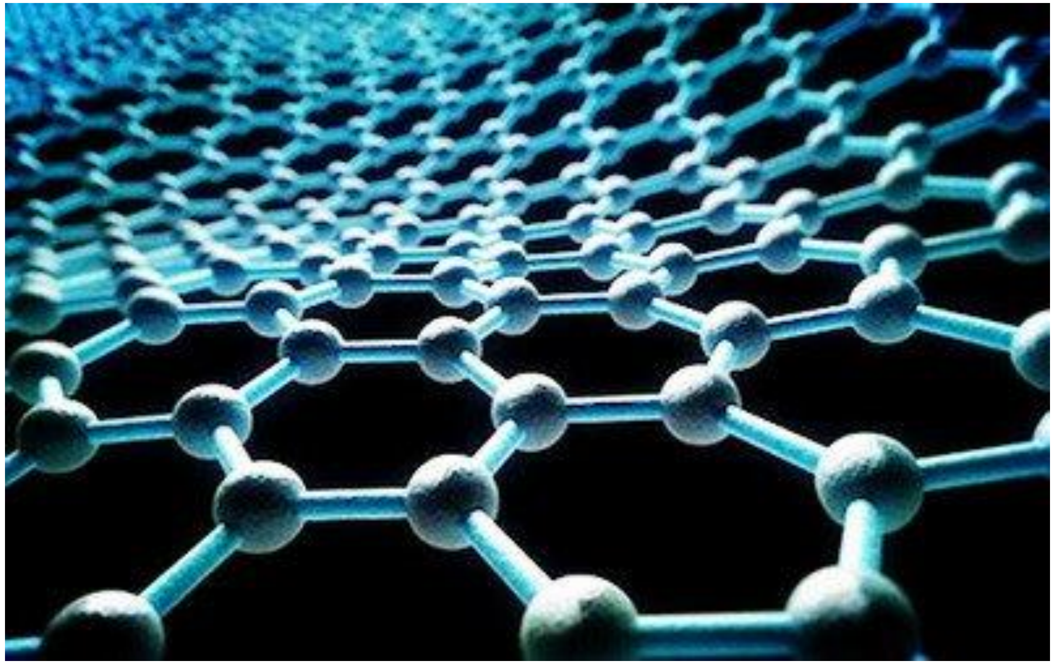


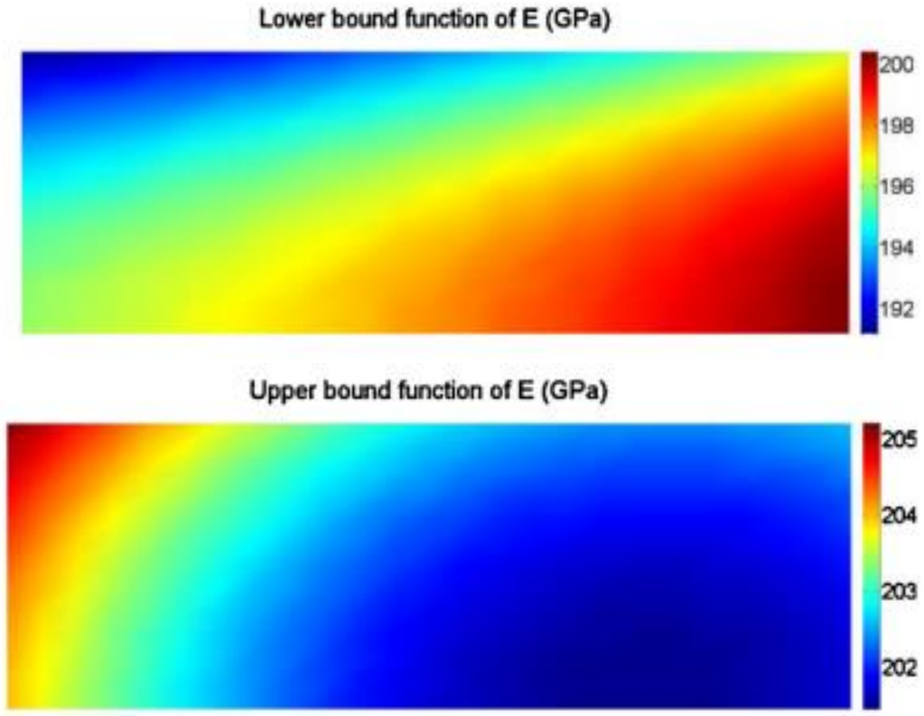


- However, the available information for determining probability density function in composites is commonly limited, the results of probabilistic analysis could be insufficiently correct.

Introduction

- When bounds of the uncertainty variables can be fixed, interval method can be used to evaluate the uncertainty effect on material performances.
- Studies on interval analysis on FG-GPLRC materials is still few, and effects of parameter uncertainty on behaviors of FG-GPLRC are unclear.
- In this work, the free vibration of an FG-GPLRC plate with interval parameters is investigated.





Free vibration of FG-GPLRC plate

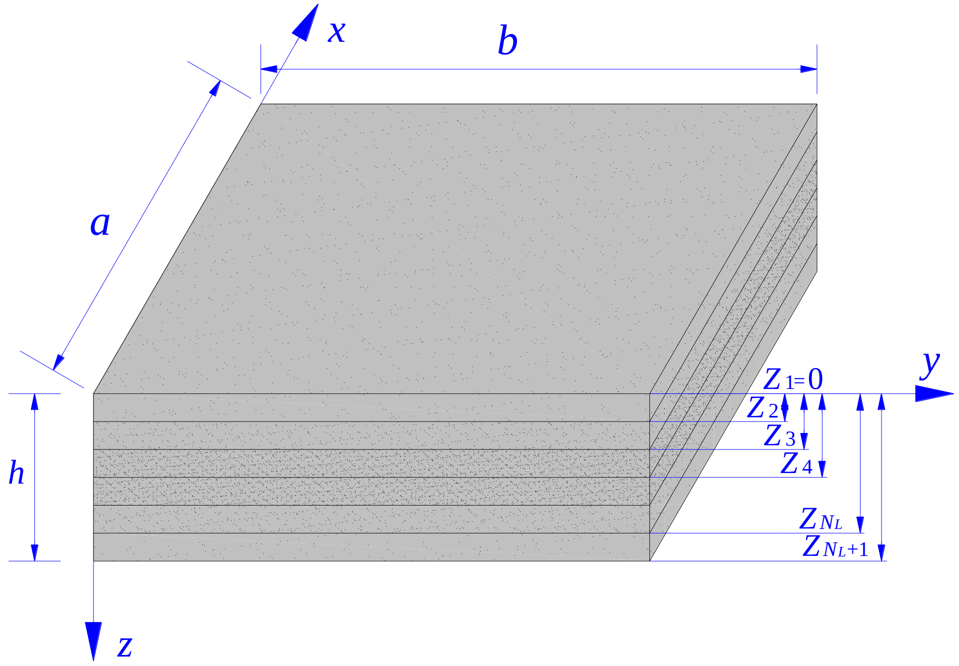
- Modified Halpin-Tsai model is adopted to obtain the effective Young's modulus of each layer of the FG-GPLRC plate:

$$E_C = \frac{3(1+\xi_L \eta_L V_{GPL})}{8(1-\eta_L V_{GPL})} E_M + \frac{5(1+\xi_W \eta_W V_{GPL})}{8(1-\eta_W V_{GPL})} E_M$$

- Mass density and Poisson's ratio of each layer can be calculated by rule of mixture :

$$\rho_C = \rho_{GPL} V_{GPL} + \rho_M V_M$$

$$v_C = v_{GPL} V_{GPL} + v_M V_M$$



A multi-layer functionally graded graphene platelets reinforced polymer plate

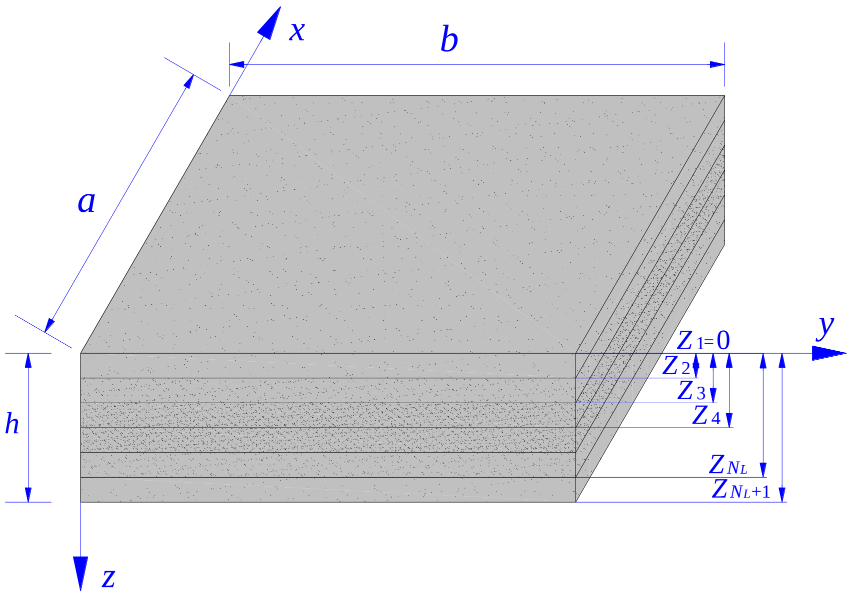
Free vibration of FG-GPLRC plate

- Use the theory of first-order shear deformation plate to setup constitutive relationship of FG-GPLRC plate :

$$U(x,y,z,t)=u_0(x,y,t)+z\phi_x(x,y,t)$$
$$V(x,y,z,t)=v_0(x,y,t)+z\phi_y(x,y,t)$$
$$W(x,y,z,t)=w_0(x,y,t)$$

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix}=\begin{Bmatrix}\dfrac{\partial u_0}{\partial x}\\ \dfrac{\partial v_0}{\partial y}\\ \dfrac{\partial u_0}{\partial y}+\dfrac{\partial v_0}{\partial x}\end{Bmatrix}+z\begin{Bmatrix}\dfrac{\partial \phi_x}{\partial x}\\ \dfrac{\partial \phi_y}{\partial y}\\ \dfrac{\partial \phi_x}{\partial y}+\dfrac{\partial \phi_y}{\partial x}\end{Bmatrix}=\boldsymbol{\varepsilon}_0+z\boldsymbol{\kappa}\qquad \begin{Bmatrix}\gamma_{yz}\\ \gamma_{xz}\end{Bmatrix}=\begin{Bmatrix}\phi_y+\dfrac{\partial w_0}{\partial y}\\ \phi_x+\dfrac{\partial w_0}{\partial x}\end{Bmatrix}=\boldsymbol{\gamma}_0$$

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \sigma_{xy}\\ \sigma_{yz}\\ \sigma_{xz}\end{Bmatrix}^{(k)}=\begin{bmatrix}\mathbf{P} & \mathbf{0}\\ \mathbf{0} & \mathbf{Q}\end{bmatrix}^{(k)}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\\ \gamma_{yz}\\ \gamma_{xz}\end{Bmatrix}^{(k)}$$



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Free vibration of FG-GPLRC plate

- The relationship between internal forces and strains can be further setup:

$$\begin{Bmatrix} \mathbf{N} \\ \mathbf{M} \end{Bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B} & \mathbf{D} \end{bmatrix} \begin{Bmatrix} \boldsymbol{\varepsilon}_0 \\ \boldsymbol{\kappa} \end{Bmatrix} \qquad \mathbf{Q} = k_s \mathbf{A}^s \boldsymbol{\gamma}_0$$

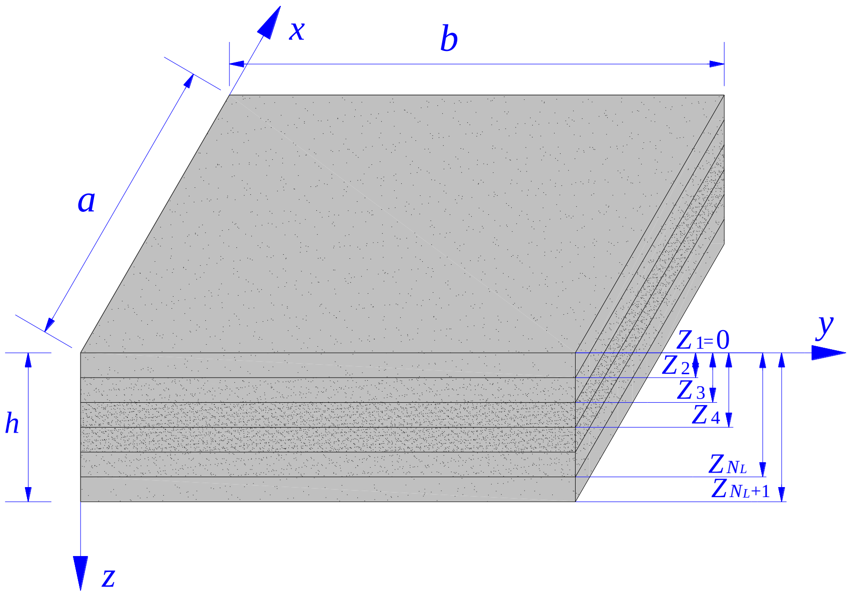
$$(A_{ij}, B_{ij}, D_{ij}) = \sum_{k=1}^{N_L} \int_{z_k}^{z_{k+1}} P_{ij}^{(k)} (1, z, z^2) \mathrm{d}z, (i, j = 1,2,3) \qquad A_{ij}^s = \sum_{k=1}^{N_L} \int_{z_k}^{z_{k+1}} Q_{ij}^{(k)} \mathrm{d}z, (i, j = 1,2)$$

- According to the Hamilton's principle, the governing equations of motion of the FG-GPLRC plate can be derived:

$$\boxed{\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = I_0 \frac{\partial^2 u_0}{\partial t^2} + I_1 \frac{\partial^2 \phi_x}{\partial t^2}} \qquad \boxed{\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = I_0 \frac{\partial^2 v_0}{\partial t^2} + I_1 \frac{\partial^2 \phi_y}{\partial t^2}}$$

$$\boxed{\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + \frac{\partial}{\partial x}\left(N_x \frac{\partial w_0}{\partial x} + N_{xy} \frac{\partial w_0}{\partial y}\right) + \frac{\partial}{\partial y}\left(N_{xy} \frac{\partial w_0}{\partial x} + N_y \frac{\partial w_0}{\partial y}\right) = I_0 \frac{\partial^2 w_0}{\partial t^2}}$$

$$\boxed{\frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} - Q_x = I_2 \frac{\partial^2 \phi_x}{\partial t^2} + I_1 \frac{\partial^2 u_0}{\partial t^2}} \qquad \boxed{\frac{\partial M_{xy}}{\partial x} + \frac{\partial M_y}{\partial y} - Q_y = I_2 \frac{\partial^2 \phi_y}{\partial t^2} + I_1 \frac{\partial^2 v_0}{\partial t^2}}$$



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◆ Free vibration of FG-GPLRC plate

- Navier's method is used to solved the above governing equation of motion:

$$u_0(x,y,t)=\sum_{m=1}^{M}\sum_{n=1}^{N}U_{mn}(t)\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b}$$

$$v_0(x,y,t)=\sum_{m=1}^{M}\sum_{n=1}^{N}V_{mn}(t)\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b}$$

$$w_0(x,y,t)=\sum_{m=1}^{M}\sum_{n=1}^{N}W_{mn}(t)\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}$$

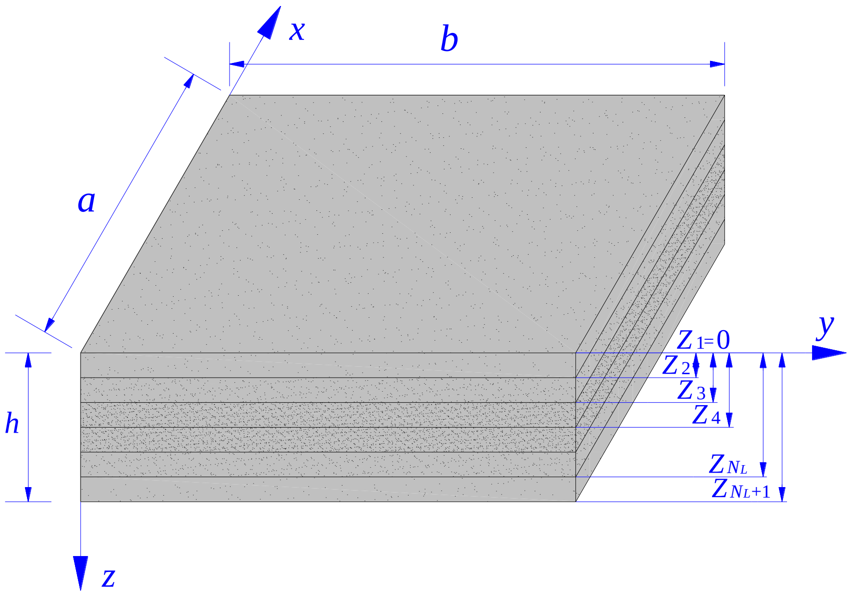
$$\phi_x(x,y,t)=\sum_{m=1}^{M}\sum_{n=1}^{N}\Phi_{x,mn}(t)\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b}$$

$$\phi_y(x,y,t)=\sum_{m=1}^{M}\sum_{n=1}^{N}\Phi_{y,mn}(t)\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b}$$

- for the simply-supported boundary conditions at all edges:

$$v_0=w_0=\varphi_y=0 \quad (x=0,a)$$

$$u_0=w_0=\varphi_x=0 \quad (y=0,b)$$



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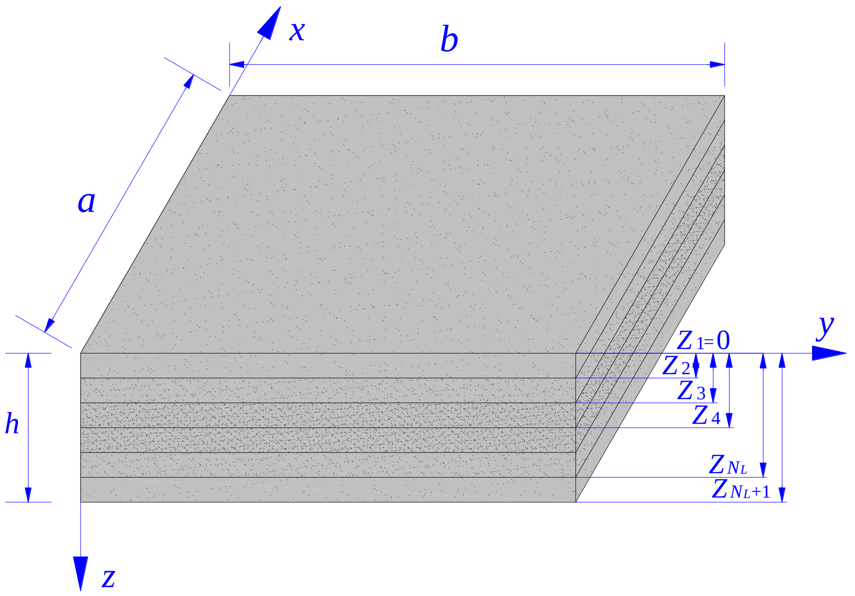
◆ Free vibration of FG-GPLRC plate

- Substitute the above Navier's displacement functions into the governing equation of motion, depending on the characteristics of double-Fourier functions, the discretized equation of free vibration of the plate can be obtained (generalized eigenvalue problem):

$$\mathbf{M}\ddot{\boldsymbol{\Delta}}(t)+\mathbf{K}\boldsymbol{\Delta}(t)=\mathbf{0} \xrightarrow{\boldsymbol{\Delta}(t)=\boldsymbol{\Delta}e^{i\omega t}} \boxed{\left(\mathbf{K}-\omega^2\mathbf{M}\right)\boldsymbol{\Delta}=\mathbf{0}}$$

$$\mathbf{M}=\begin{bmatrix} I_0 & 0 & 0 & I_1 & 0 \\ 0 & I_0 & 0 & 0 & I_1 \\ 0 & 0 & I_0 & 0 & 0 \\ I_1 & 0 & 0 & I_2 & 0 \\ 0 & I_1 & 0 & 0 & I_2 \end{bmatrix} \qquad \mathbf{K}=\begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} & k_{15} \\ k_{21} & k_{22} & k_{23} & k_{24} & k_{25} \\ k_{31} & k_{32} & k_{33} & k_{34} & k_{35} \\ k_{41} & k_{42} & k_{43} & k_{44} & k_{45} \\ k_{51} & k_{52} & k_{53} & k_{54} & k_{55} \end{bmatrix}$$

$$\boldsymbol{\Delta}(t)=\left[U_{mn}(t),V_{mn}(t),W_{mn}(t),\Phi_{x,mn}(t),\Phi_{y,mn}(t)\right]^{\mathrm{T}}$$



A multi-layer functionally graded graphene platelets reinforced polymer plate

Interval eigenvalue analysis

- In the non-probability interval framework, an uncertain quantity is expressed its upper and lower bounds:

$$\alpha_i^I = [\underline{\alpha}_i, \overline{\alpha}_i]$$

- In engineering analysis, an interval variable is commonly denoted by its midpoint and the uncertainty degree (rate of deviation radius to midpoint):

$$\alpha_i^{\mathrm{C}} = \frac{\underline{\alpha}_i + \overline{\alpha}_i}{2} \qquad \Delta\alpha_i = \frac{(\overline{\alpha}_i - \underline{\alpha}_i)}{2\alpha_i^C} > 0$$

- An interval vector or matrix is who includes interval parameters in its elements:

$$\boldsymbol{\alpha}^I = \left\{\alpha_1^I, \alpha_2^I, \cdots, \alpha_n^I\right\}$$

Interval eigenvalue analysis

- Therefore, for a free vibration problem with interval parameters, the corresponding mathematical model is a generalized interval eigenvalue problem:

Interval vectors

$$\mathbf{K}(\boldsymbol{\alpha})\phi(\boldsymbol{\alpha}) = \lambda(\boldsymbol{\alpha})\mathbf{M}(\boldsymbol{\alpha})\phi(\boldsymbol{\alpha}) \qquad \boldsymbol{\alpha} \in \boldsymbol{\alpha}^I = [\underline{\boldsymbol{\alpha}}, \overline{\boldsymbol{\alpha}}] \qquad \lambda(\boldsymbol{\alpha}^I) = \omega^2(\boldsymbol{\alpha}^I)$$

Interval matrices

- In engineering interval problems, the phenomenon of interval dependency may affect the accuracy of interval bounds of structural performances.
- Interval dependency: For an uncertain physical quantity in the governing equation of engineering problem, it should take the same value anywhere it appears in the governing equation. But in interval computations, it may take different values in different places where it appears in the corresponding interval form of the governing equation. Such a phenomenon could make the bounds of solutions be exaggerated.
- Therefore, in order to achieve real bounds of the above interval eigenvalues, an effective method should be found to overcome the phenomenon of interval dependency.
- Here, we turn to evaluate the monotony of eigenvalues on the interval variables, which could be a potential direction. If the monotony exists, the interval variable will take the same bound anywhere it appears. Thus, interval dependency disappear automatically.

Interval eigenvalue analysis

- Consequently, we differentiate the above generalized eigenvalue problem on each interval variable:

$$\frac{\partial \mathbf{K}(\boldsymbol{\alpha})}{\partial \alpha_i}\phi(\boldsymbol{\alpha})+\mathbf{K}(\boldsymbol{\alpha})\frac{\partial \phi(\boldsymbol{\alpha})}{\partial \alpha_i}=\frac{\partial \lambda(\boldsymbol{\alpha})}{\partial \alpha_i}\mathbf{M}(\boldsymbol{\alpha})\phi(\boldsymbol{\alpha})+\lambda(\boldsymbol{\alpha})\frac{\partial \mathbf{M}(\boldsymbol{\alpha})}{\partial \alpha_i}\phi(\boldsymbol{\alpha})+\lambda(\boldsymbol{\alpha})\mathbf{M}(\boldsymbol{\alpha})\frac{\partial \phi(\boldsymbol{\alpha})}{\partial \alpha_i}$$

- Further, combine similar terms:

$$\left[\frac{\partial \mathbf{K}(\boldsymbol{\alpha})}{\partial \alpha_i}-\frac{\partial \lambda(\boldsymbol{\alpha})}{\partial \alpha_i}\mathbf{M}(\boldsymbol{\alpha})-\lambda(\boldsymbol{\alpha})\frac{\partial \mathbf{M}(\boldsymbol{\alpha})}{\partial \alpha_i}\right]\phi(\boldsymbol{\alpha})+\left[\mathbf{K}(\boldsymbol{\alpha})-\lambda(\boldsymbol{\alpha})\mathbf{M}(\boldsymbol{\alpha})\right]\frac{\partial \phi(\boldsymbol{\alpha})}{\partial \alpha_i}=0$$

- A transposition of eigenvector can be right-multiplied on the equation:

$$\phi^{\mathrm{T}}(\boldsymbol{\alpha})\left[\frac{\partial \mathbf{K}(\boldsymbol{\alpha})}{\partial \alpha_i}-\frac{\partial \lambda(\boldsymbol{\alpha})}{\partial \alpha_i}\mathbf{M}(\boldsymbol{\alpha})-\lambda(\boldsymbol{\alpha})\frac{\partial \mathbf{M}(\boldsymbol{\alpha})}{\partial \alpha_i}\right]\phi(\boldsymbol{\alpha})+\phi^{\mathrm{T}}(\boldsymbol{\alpha})\left[\mathbf{K}(\boldsymbol{\alpha})-\lambda(\boldsymbol{\alpha})\mathbf{M}(\boldsymbol{\alpha})\right]\frac{\partial \phi(\boldsymbol{\alpha})}{\partial \alpha_i}=0$$

Interval eigenvalue analysis

- At the same time, the eigenvalue equation can be transformed as:

$$\left(\mathbf{K}(\boldsymbol{\alpha})\phi(\boldsymbol{\alpha})\right)^{T}=\left(\lambda(\boldsymbol{\alpha})\mathbf{M}(\boldsymbol{\alpha})\phi(\boldsymbol{\alpha})\right)^{T}$$

Expand

$$\phi^{\mathrm{T}}(\boldsymbol{\alpha})\mathbf{K}^{\mathrm{T}}(\boldsymbol{\alpha})=\lambda(\boldsymbol{\alpha})\phi^{\mathrm{T}}(\boldsymbol{\alpha})\mathbf{M}^{\mathrm{T}}(\boldsymbol{\alpha})$$

Combine similar terms

$$\phi^{\mathrm{T}}(\boldsymbol{\alpha})\left(\mathbf{K}^{\mathrm{T}}(\boldsymbol{\alpha})-\lambda(\boldsymbol{\alpha})\mathbf{M}^{\mathrm{T}}(\boldsymbol{\alpha})\right)=0$$

Symmetry-mass/stiffness matrix

$$\phi^{\mathrm{T}}(\boldsymbol{\alpha})\left(\mathbf{K}(\boldsymbol{\alpha})-\lambda(\boldsymbol{\alpha})\mathbf{M}(\boldsymbol{\alpha})\right)=0$$

◆ Interval eigenvalue analysis

$$\phi^{\mathrm{T}}(\boldsymbol{\alpha})\left(\mathbf{K}(\boldsymbol{\alpha})-\lambda(\boldsymbol{\alpha})\mathbf{M}(\boldsymbol{\alpha})\right)=0$$

$$\phi^{\mathrm{T}}(\boldsymbol{\alpha})\left[\frac{\partial\mathbf{K}(\boldsymbol{\alpha})}{\partial\alpha_i}-\frac{\partial\lambda(\boldsymbol{\alpha})}{\partial\alpha_i}\mathbf{M}(\boldsymbol{\alpha})-\lambda(\boldsymbol{\alpha})\frac{\partial\mathbf{M}(\boldsymbol{\alpha})}{\partial\alpha_i}\right]\phi(\boldsymbol{\alpha})+\boxed{\phi^{\mathrm{T}}(\boldsymbol{\alpha})\left[\mathbf{K}(\boldsymbol{\alpha})-\lambda(\boldsymbol{\alpha})\mathbf{M}(\boldsymbol{\alpha})\right]\frac{\partial\phi(\boldsymbol{\alpha})}{\partial\alpha_i}}=0$$

$$\phi^{\mathrm{T}}(\boldsymbol{\alpha})\left[\frac{\partial\mathbf{K}(\boldsymbol{\alpha})}{\partial\alpha_i}-\frac{\partial\lambda(\boldsymbol{\alpha})}{\partial\alpha_i}\mathbf{M}(\boldsymbol{\alpha})-\lambda(\boldsymbol{\alpha})\frac{\partial\mathbf{M}(\boldsymbol{\alpha})}{\partial\alpha_i}\right]\phi(\boldsymbol{\alpha})=0$$

The interval variable takes its midpoint

$$\phi_0^{\mathrm{T}}\left[\left.\frac{\partial\mathbf{K}(\boldsymbol{\alpha})}{\partial\alpha_i}\right|_{\boldsymbol{\alpha}=\boldsymbol{\alpha}^{\mathrm{C}}}-\left.\frac{\partial\lambda(\boldsymbol{\alpha})}{\partial\alpha_i}\right|_{\boldsymbol{\alpha}=\boldsymbol{\alpha}^{\mathrm{C}}}\mathbf{M}_0-\lambda_0\left.\frac{\partial\mathbf{M}(\boldsymbol{\alpha})}{\partial\alpha_i}\right|_{\boldsymbol{\alpha}=\boldsymbol{\alpha}^{\mathrm{C}}}\right]\phi_0=0$$

$$\phi_0^{\mathrm{T}}\mathbf{M}_0\phi_0=1$$

Sensitivity of any eigenvalue to any interval variable (Sofi et al., 2015):

$$S_{\alpha_i}=\left.\frac{\partial\lambda(\boldsymbol{\alpha})}{\partial\alpha_i}\right|_{\boldsymbol{\alpha}=\boldsymbol{\alpha}^{\mathrm{C}}}=\phi_0^{\mathrm{T}}\left.\frac{\partial\mathbf{K}(\boldsymbol{\alpha})}{\partial\alpha_i}\right|_{\boldsymbol{\alpha}=\boldsymbol{\alpha}^{\mathrm{C}}}\phi_0-\lambda_0\phi_0^{\mathrm{T}}\left.\frac{\partial\mathbf{M}(\boldsymbol{\alpha})}{\partial\alpha_i}\right|_{\boldsymbol{\alpha}=\boldsymbol{\alpha}^{\mathrm{C}}}\phi_0$$

Interval eigenvalue analysis

- Bounds of intervals used for computation is decided by the results of sensitivity:

$S_{\alpha_i} > 0$ → Eigenvalue is increasing function of the interval → $\alpha_i^{(\mathrm{LB})} = \underline{\alpha}_i$ $\alpha_i^{(\mathrm{UB})} = \overline{\alpha}_i$

$S_{\alpha_i} < 0$ → Eigenvalue is decreasing function of the interval → $\alpha_i^{(\mathrm{LB})} = \overline{\alpha}_i$ $\alpha_i^{(\mathrm{UB})} = \underline{\alpha}_i$

- The interval eigenvalue problem can be evaluated by two deterministic eigenvalue problems:

$$\mathbf{K}(\boldsymbol{\alpha}^{(\mathrm{LB})})\phi(\boldsymbol{\alpha}^{(\mathrm{LB})}) = \underline{\lambda}(\boldsymbol{\alpha}^{(\mathrm{LB})})\mathbf{M}(\boldsymbol{\alpha}^{(\mathrm{LB})})\phi(\boldsymbol{\alpha}^{(\mathrm{LB})})$$
$$\mathbf{K}(\boldsymbol{\alpha}^{(\mathrm{UB})})\phi(\boldsymbol{\alpha}^{(\mathrm{LB})}) = \overline{\lambda}(\boldsymbol{\alpha}^{(\mathrm{LB})})\mathbf{M}(\boldsymbol{\alpha}^{(\mathrm{UB})})\phi(\boldsymbol{\alpha}^{(\mathrm{LB})})$$

Solve deterministic eigenvalue problem when intervals take their midpoints

↓

Compute derivatives of matrices of mass and stiffness on intervals when they take midpoints

↓

Obtain sensitivities of eigenvalues on intervals

↓

Determine bounds of intervals corresponding to the bounds of eigenvalues

↓

Compute bounds of eigenvalues by solving two deterministic eigenvalue problems

$$\mathbf{K}(\boldsymbol{\alpha})\boxed{\phi_0(\boldsymbol{\alpha})}=\boxed{\lambda_0(\boldsymbol{\alpha})}\mathbf{M}(\boldsymbol{\alpha})\phi_0(\boldsymbol{\alpha})$$

$$S_{\alpha_i}=\left.\frac{\partial\lambda(\boldsymbol{\alpha})}{\partial\alpha_i}\right|_{\boldsymbol{\alpha}=\boldsymbol{\alpha}^{\mathrm{C}}}=\phi_0^{\mathrm{T}}\boxed{\left.\frac{\partial\mathbf{K}(\boldsymbol{\alpha})}{\partial\alpha_i}\right|_{\boldsymbol{\alpha}=\boldsymbol{\alpha}^{\mathrm{C}}}}\phi_0-\lambda_0\phi_0^{\mathrm{T}}\boxed{\left.\frac{\partial\mathbf{M}(\boldsymbol{\alpha})}{\partial\alpha_i}\right|_{\boldsymbol{\alpha}=\boldsymbol{\alpha}^{\mathrm{C}}}}\phi_0$$

$$S_{\alpha_i}>0 \longrightarrow \alpha_i^{(\mathrm{LB})}=\underline{\alpha}_i \qquad \alpha_i^{(\mathrm{UB})}=\overline{\alpha}_i$$

$$S_{\alpha_i}<0 \longrightarrow \alpha_i^{(\mathrm{LB})}=\overline{\alpha}_i \qquad \alpha_i^{(\mathrm{UB})}=\underline{\alpha}_i$$

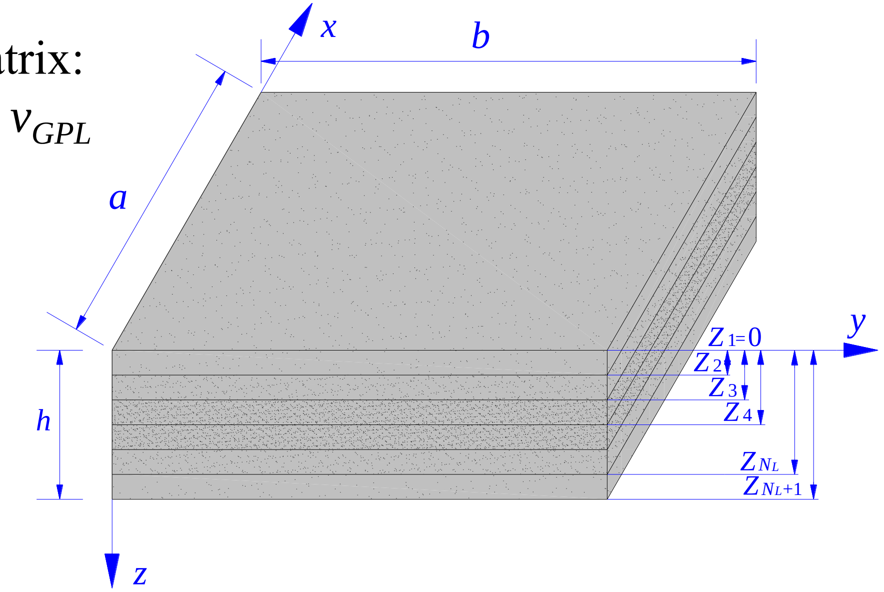
$$\mathbf{K}(\boldsymbol{\alpha}^{(\mathrm{LB})})\phi(\boldsymbol{\alpha}^{(\mathrm{LB})})=\underline{\lambda}(\boldsymbol{\alpha}^{(\mathrm{LB})})\mathbf{M}(\boldsymbol{\alpha}^{(\mathrm{LB})})\phi(\boldsymbol{\alpha}^{(\mathrm{LB})})$$

$$\mathbf{K}(\boldsymbol{\alpha}^{(\mathrm{UB})})\phi(\boldsymbol{\alpha}^{(\mathrm{LB})})=\overline{\lambda}(\boldsymbol{\alpha}^{(\mathrm{LB})})\mathbf{M}(\boldsymbol{\alpha}^{(\mathrm{UB})})\phi(\boldsymbol{\alpha}^{(\mathrm{LB})})$$

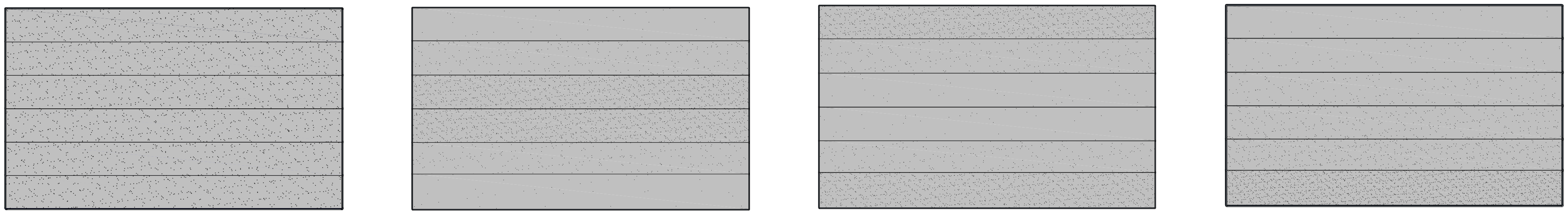
Procedure of interval free vibration analysis of FG-GPLRC plates

Results and discussions

- width × length × thickness of plate: $a \times b \times h$ = 0.45m×0.45m×0.045m
- Average dimension of GPLs: $l_{GPL} \times w_{GPL} \times h_{GPL}$ = 2.5μm×1.5μm×1.5nm
- Density, Young's moduls and Poisson's ratio of GPLs and polymer matrix: $\rho_{GPL} = 1.06\text{g}/\text{cm}^3$，$\rho_M = 1.2\text{g}/\text{cm}^3$，$E_{GPL} = 1.01\text{TPa}$，$E_M = 3\text{GPa}$，$v_{GPL} = 0.186$和$v_M = 0.34$
- Weight fraction of GPLs in plate: $g_{GPL} = 1\%$
- Total layers: $N_L = 10$



$$\boxed{E_C^{(k)^I} = E_C^{(k)^C}(1+\Delta\alpha) \quad \Delta\alpha=0.1}$$



Four patterns of GPLs distribution (linear variation among layers)

◆ Results and discussions

$$\frac{\partial \mathbf{K}}{\partial E_C^{(k)}} = \frac{\partial}{\partial E_C^{(k)}} \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} & k_{15} \\ k_{21} & k_{22} & k_{23} & k_{24} & k_{25} \\ k_{31} & k_{32} & k_{33} & k_{34} & k_{35} \\ k_{41} & k_{42} & k_{43} & k_{44} & k_{45} \\ k_{51} & k_{52} & k_{53} & k_{54} & k_{55} \end{bmatrix}$$

$$\frac{\partial k_{11}}{\partial E_C^{(k)}} = \pi^2 \left(\frac{m^2}{a^2} \boxed{\frac{\partial A_{11}}{\partial E_C^{(k)}}} + \frac{n^2}{b^2} \frac{\partial A_{33}}{\partial E_C^{(k)}} \right) \qquad \frac{\partial k_{14}}{\partial E_C^{(k)}} = \pi^2 \left(\frac{m^2}{a^2} \boxed{\frac{\partial B_{11}}{\partial E_C^{(k)}}} + \frac{n^2}{b^2} \frac{\partial B_{33}}{\partial E_C^{(k)}} \right)$$

$$\frac{\partial k_{43}}{\partial E_C^{(k)}} = \pi k_s \frac{m}{a} \boxed{\frac{\partial A_{22}^s}{\partial E_C^{(k)}}} \qquad \frac{\partial k_{44}}{\partial E_C^{(k)}} = \frac{m^2 \pi^2}{a^2} \boxed{\frac{\partial D_{11}}{\partial E_C^{(k)}}} + \frac{n^2 \pi^2}{b^2} \frac{\partial D_{33}}{\partial E_C^{(k)}} + k_s \frac{\partial A_{22}^s}{\partial E_C^{(k)}}$$

$$\frac{\partial A_{ij}}{\partial E_c^{(k)}} = \frac{z_{k+1} - z_k}{1 - (v_c^{(k)})^2} \begin{bmatrix} 1 & v_c^{(k)} & 0 \\ v_c^{(k)} & 1 & 0 \\ 0 & 0 & \dfrac{1 - v_c^{(k)}}{2} \end{bmatrix} \qquad \frac{\partial B_{ij}}{\partial E_c^{(k)}} = \frac{z_{k+1}^2 - z_k^2}{2 \times [1 - (v_c^{(k)})^2]} \begin{bmatrix} 1 & v_c^{(k)} & 0 \\ v_c^{(k)} & 1 & 0 \\ 0 & 0 & \dfrac{1 - v_c^{(k)}}{2} \end{bmatrix}$$

$$\frac{\partial D_{ij}}{\partial E_c^{(k)}} = \frac{z_{k+1}^3 - z_k^3}{3 \times [1 - (v_c^{(k)})^2]} \begin{bmatrix} 1 & v_c^{(k)} & 0 \\ v_c^{(k)} & 1 & 0 \\ 0 & 0 & \dfrac{1 - v_c^{(k)}}{2} \end{bmatrix} \qquad \frac{\partial A_{ij}^s}{\partial E_c^{(k)}} = \frac{z_{k+1} - z_k}{2(1 + v_c^{(k)})} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Results and discussions

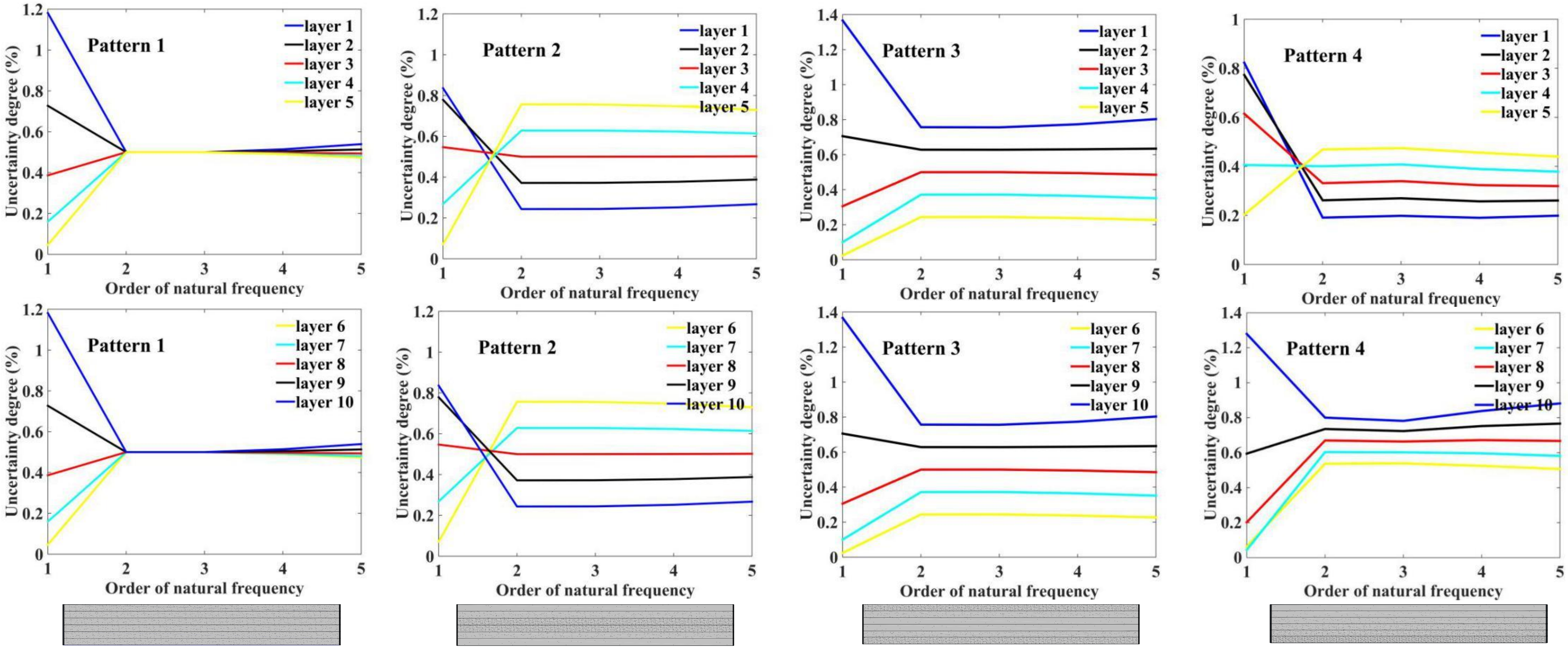
- All sensitivities are positive for any pattern
- Bounds of eigenvalue are consistent with bounds of interval parameters

Pattern 3: Sensitivities of first five eigenvalues on Young's modulus of each layer

λ_j	$E_C^{(1)}$	$E_C^{(2)}$	$E_C^{(3)}$	$E_C^{(4)}$	$E_C^{(5)}$	$E_C^{(6)}$	$E_C^{(7)}$	$E_C^{(8)}$	$E_C^{(9)}$	$E_C^{(10)}$
$j=1$	+	+	+	+	+	+	+	+	+	+
$j=2$	+	+	+	+	+	+	+	+	+	+
$j=3$	+	+	+	+	+	+	+	+	+	+
$j=4$	+	+	+	+	+	+	+	+	+	+
$j=5$	+	+	+	+	+	+	+	+	+	+

◆ Results and discussions

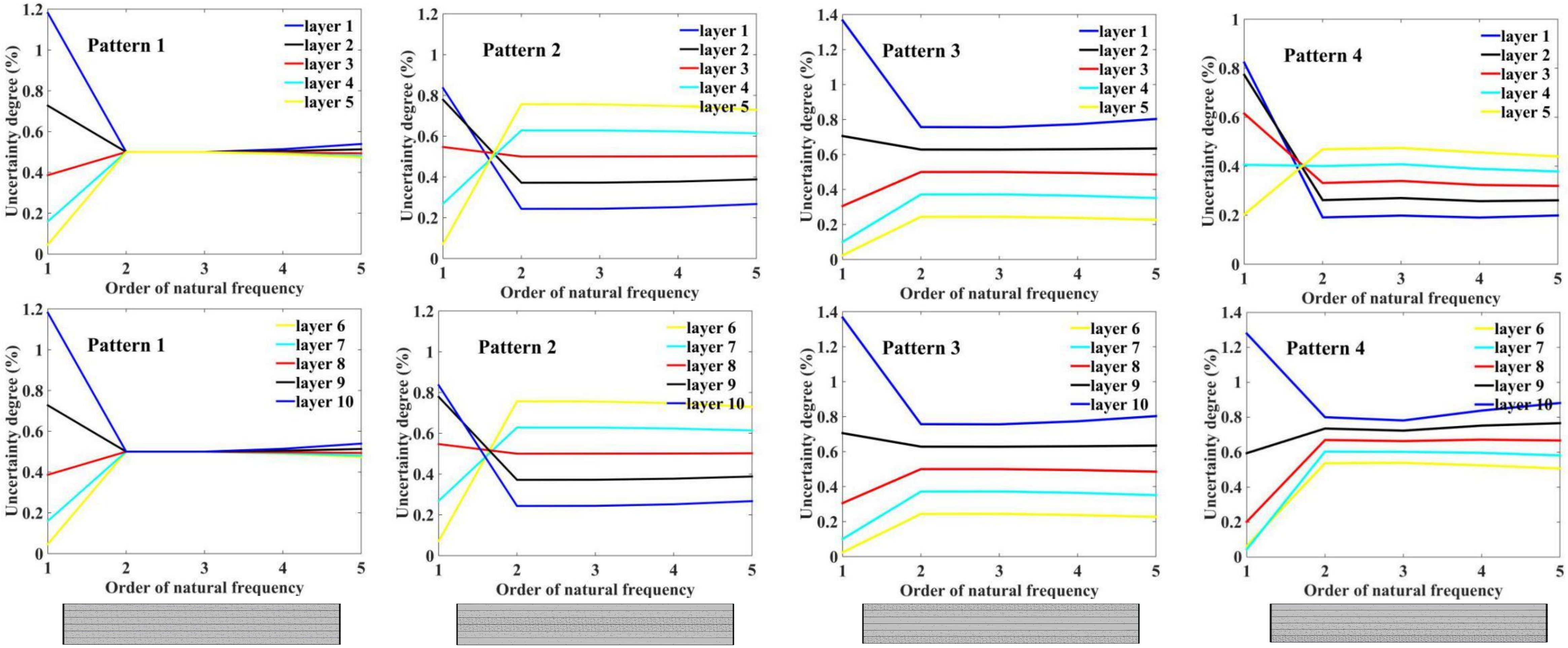
- The effect of uncertainty of Young’s modulus is different among patterns
- The effect is different among orders of eigenvalue, but basically invariant after 2^{nd} order of eigenvalue



Effect of uncertainty of any individual layer on first five eigenvalues

◆ Results and discussions

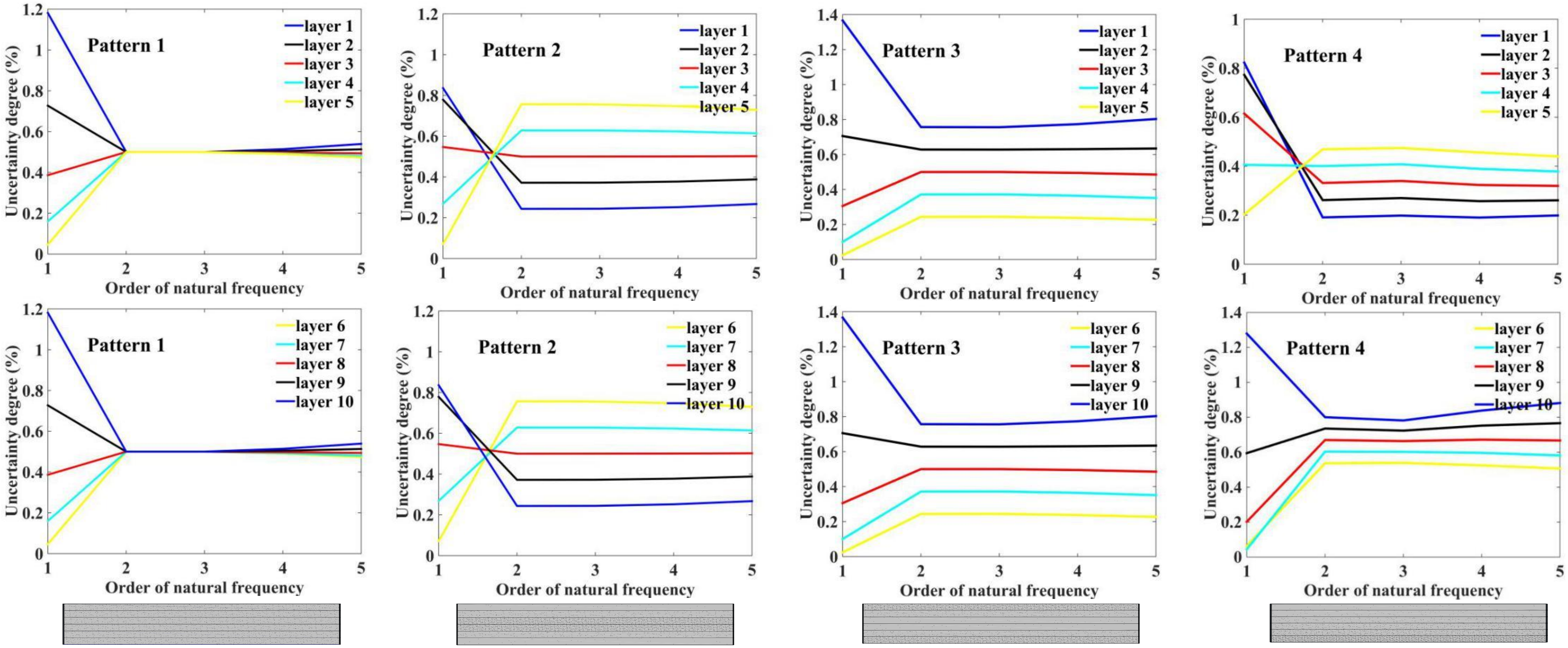
- The number of uncertain layer obviously influent the effect of uncertainty:
  - ✓ For 1st eigenvalue, the effect from the surface layer (1st and 10th) is more significant (no matter patterns)
  - ✓ For eigenvalues after 2nd, it is the layer with more GPLs produces more obvious effect of uncertainty



Effect of uncertainty of any individual layer on first five eigenvalues

Results and discussions

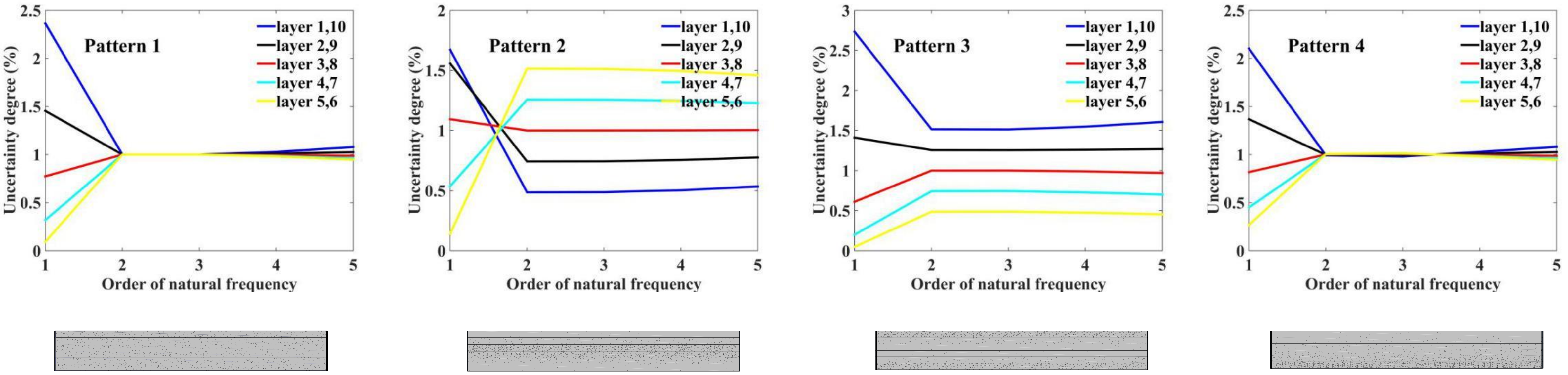
- The layer with same GPLs weight produces same uncertainty of eigenvalue (Symmetry)
- Pattern 3 has the largest uncertainty degree of eigenvalue



Effect of uncertainty of any individual layer on first five eigenvalues

Results and discussions

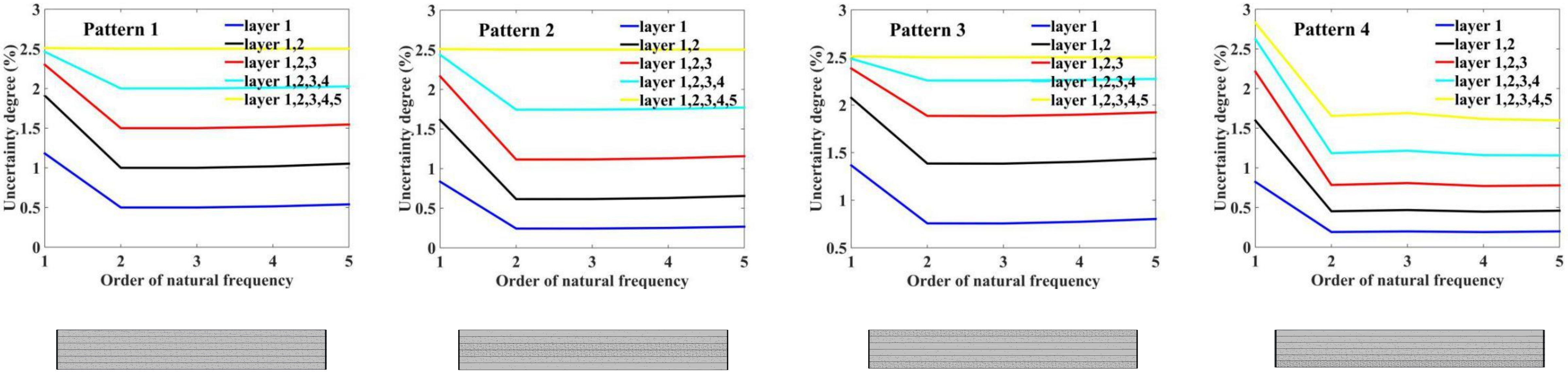
- ✓ The effects of two layers are linearly superposed (Shape of curves don’t change)
- ✓ Pattern 4, spatially symmetric layers have different GPLs weight, so the effect keeps invariant after superposition



Effect of uncertainty when both layers of opposite positions are interval

◆ Results and discussions

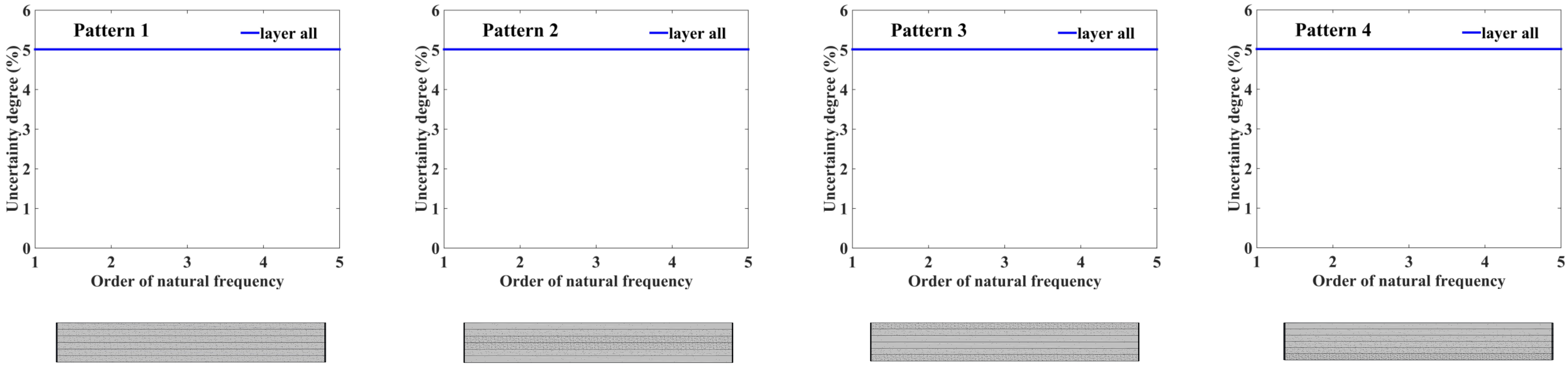
- ✓ With the increasing of uncertain layers, the uncertainty degree of each eigenvalue gradually increases
- ✓ For patterns with symmetric distribution-GPLs, uncertainty under all 5 uncertain layers becomes same for all eigenvalues



Variation of uncertainty effect under increasing number of interval layers

Results and discussions

- ✓ When all the 10 layers are uncertain, the uncertainty degree for all patterns are 5%
- ✓ Although the u.c. of eigenvalue is lower than that of Young's modulus (10%), when more types of material parameters are uncertain, u.c. of eigenvalue could be expected to exceed 10%



Uncertainty degree of eigenvalues when all layers are interval

Conclusions

- Uncertainty of Young's modulus results in uncertainty of natural frequencies of FG-GPLRC plate
- Under same uncertainty degree of Young's modulus, uncertainty degree of 1st eigenvalue is different from that of higher orders of eigenvalues
- Uncertainty effect differs among GPLs patterns, with the most eminent effect in Pattern 3
- For 1st eigenvalue (fundamental frequency), the layer near plate surface has more obvious uncertainty effect
- For other eigenvalues, the effect from the layer with more GPLs is more significant

For all patterns

- The layers with symmetric spatial position have the same effect, and the effect of layers can be linearly superposed. → For all eigenvalues
- The uncertainty degree of all natural frequencies of FG-GPLRC plate are less than the uncertainty degree of Young's modulus.

THANKS!