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Serviceability assessment of footbridges via interval analysis

Roberta Santoro (University of Messina)
Alba Sofi (University of Reggio Calabria)
Federica Tubino (University of Genoa)

Motivation and Objectives

Serviceability assessment is a critical requirement for modern footbridges:

- Guidelines provide conventional equivalent loading models (e.g. uniformly distributed resonant loads, SETRA 2006)
- Equivalent Spectral Model allows to estimate the dynamic response as a function of loading and structural parameters (Piccardo & Tubino 2012)

Loading and structural parameters are uncertain (Lievens et al. 2016, Tubino et al. 2020)

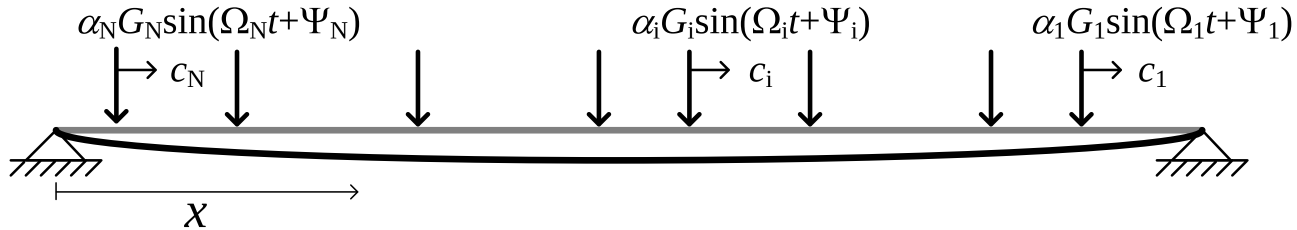
Probability distribution of uncertain parameters is not available: interval analysis (Moore, 1966) is an appropriate tool

Objective: Closed-form solution of the lower bound and upper bound of the quantities of interest and of serviceability assessment

Presentation Outline

- Analytical Formulation
- Serviceability analysis with interval uncertainties
- Closed form solution
- Numerical application
- Conclusions

Analytical Formulation (1/4)



Random variables

Equation of motion

$$m_s(x)\frac{\partial^2 q(x,t)}{\partial t^2}+\mathrm{C}\left[\frac{\partial q(x,t)}{\partial t}\right]+\mathrm{L}\left[q(x,t)\right]=f(x,t)$$

$$f(x,t)=\sum_{i=1}^{N_p}\alpha_i G_i \sin\left(\Omega_i(t-\tau_i)+\Psi_i\right)\delta\left[x-c_i(t-\tau_i)\right]\left[H(t-\tau_i)-H\left(t-\tau_i-\frac{L}{c_i}\right)\right]$$

Principal transformation

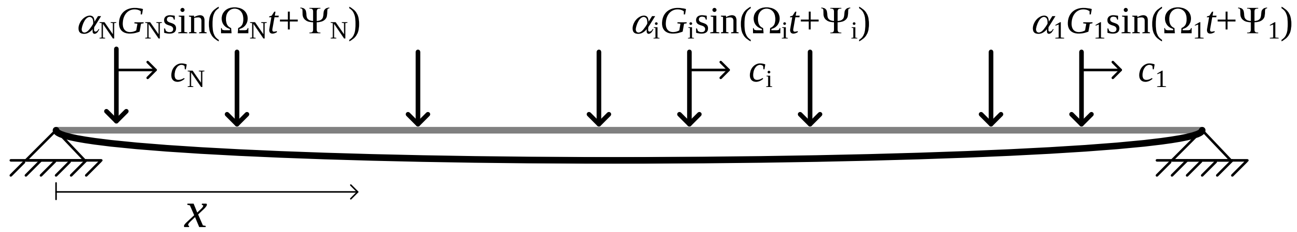
$$q(x,t)=\varphi_j(x)\,p_j(t)$$

$$\ddot{p}_j(t)+2\xi_j\omega_j\dot{p}_j(t)+\omega_j^2 p_j(t)=\frac{1}{M_j}F_j(t)$$

$$F_j(t)=\int_0^L f(x,t)\varphi_j(x)dx$$

Monte Carlo simulation

Analytical Formulation (2/4)



Equivalent spectral model (Piccardo & Tubino 2012)

Psdf of the modal force

$$S_{F_j}(\omega) = \frac{N_p}{4}(\alpha G)^2 p_\Omega(\omega)$$

Pdf of the circular step frequency

$$p_\Omega(\omega) = \frac{1}{\sigma_{\omega p}\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\omega - \mu_{\omega p}}{\sigma_{\omega p}}\right)^2\right]$$

Psdf of the acceleration of the principal coordinate

$$S_{\ddot{p}_j}(\omega) = \omega^4 \left|H_j(\omega)\right|^2 S_{F_j}(\omega)$$

Spectral Moments

$$\lambda_{0,\ddot{p}_j} = \sigma^2_{\ddot{p}_j} \approx \frac{\pi\omega_j S_{F_j}(\omega_j)}{4\xi_j M_j^2} \qquad \lambda_{2,\ddot{p}_j} \approx \frac{\pi\omega_j^3 S_{F_j}(\omega_j)}{4\xi_j M_j^2}$$

Analytical Formulation (3/4)

Maximum acceleration in [0 T]

$$\ddot{p}_{j,\max}(T) = \max_{0 \le t \le T} \left| \ddot{p}_j(t) \right| \qquad T = \frac{NL}{c_m}; \quad c_m = 0.9 \frac{\mu_{\omega p}}{2\pi}$$

Cumulative distribution function (CDF, Rice, 1944)

$$L_{\ddot{p}_{j,\max}}(b,T) = \Pr\left[\ddot{p}_{j,\max}(T) \le b \right] \approx \exp\left[-2 \ \nu_{\ddot{p}_j} T \exp\left(-\frac{b^2}{2\sigma^2_{\ddot{p}_j}} \right) \right]$$

Expected Frequency $$\nu_{\ddot{p}_j} = \frac{1}{2\pi} \sqrt{\frac{\lambda_{2,\ddot{p}_j}}{\lambda_{0,\ddot{p}_j}}} \approx \frac{\omega_j}{2\pi}$$

Mean value of the maximum acceleration (Davenport, 1964)

$$\ddot{q}_{\max} = \mathrm{E}\left[\ddot{p}_{j_{\max}} \right] = g_{\ddot{p}_j} \sigma_{\ddot{p}_j}$$

Peak factor $$g_{\ddot{p}_j} = \sqrt{2 \ln\left(2 \frac{NL}{0.9} \frac{\omega_j}{\mu_{\omega p}} \right)} + \frac{0.5772}{\sqrt{2 \ln\left(2 \frac{NL}{0.9} \frac{\omega_j}{\mu_{\omega p}} \right)}}$$

Analytical Formulation (4/4)

Serviceability assessment

Comfort classes (SETRA 2006)

Comfort Class $C^{(i)}$	a_{min} (m/s²)	a_{max} (m/s²)
Maximum (i=1)	0	0.5
Medium (i=2)	0.5	1
Minimum (i=3)	1	2.5
Unacceptable (i=4)	2.5	

Approach 1

$$\ddot{q}_{\max} = \mathrm{E}\left[\ddot{p}_{j_{\max}}\right] = g_{\ddot{p}_j}\sigma_{\ddot{p}_j} \qquad C = C^{(i)}\left[a_{\min}^{(i)} \le \ddot{q}_{\max} < a_{\max}^{(i)}\right]$$

Approach 2

$$L_{\ddot{p}_{j,\max}}(b) = \Pr\left[\ddot{p}_{j,\max} \le b\right] \approx \exp\left[-2\,\frac{NL}{0.9}\,\frac{\omega_j}{\mu_{\omega p}}\exp\left(-\frac{b^2}{2\sigma_{\ddot{p}_j}^2}\right)\right]$$

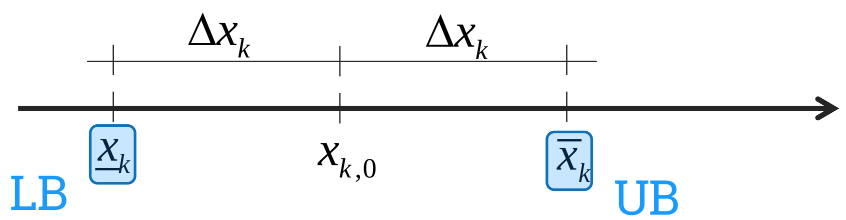
$$\Pr\left[C^{(i)}\right] = \Pr\left[\ddot{p}_{j,\max} \le a_{\max}^{(i)}\right] = L_{\ddot{p}_{j,\max}}(a_{\max}^{(i)})$$

Serviceability analysis with interval uncertainties

Interval loading and structural parameters

Classical Interval Analysis (Moore 1966)

$$\mathbf{x}^I = \left\{ \mu^I_{\omega p} \quad \sigma^I_{\omega p} \quad \alpha^I \quad G^I \quad \omega^I_j \quad \xi^I_j \quad M^I_j \right\}^T$$



$$x^I_k = \left[\underline{x}_k, \overline{x}_k \right]$$

Nominal value $x_{k,0} = \dfrac{\overline{x}_k + \underline{x}_k}{2}$

Normalized deviation amplitude $\Delta\chi_k = \dfrac{\Delta x_k}{x_{k,0}} = \dfrac{\overline{x}_k - \underline{x}_k}{2x_{k,0}}$ $\quad 0 < \Delta\chi_k < 1$

$$x^I_k = x_{k,0} + \Delta x_k e^I_k = x_{k,0}\left(1 + \Delta\chi_k e^I_k\right) \qquad e^I_k = [-1,+1]$$

Improved interval analysis with Extra Unitary Interval (Muscolino & Sofi 2012)

$$x^I_k = x_{k,0} + \Delta x_k \hat{e}^I_k = x_{k,0}\left(1 + \Delta\chi_k \hat{e}^I_k\right) \qquad \hat{e}^I_k = [-1,+1]$$

$$\hat{e}^I_k - \hat{e}^I_k = [0,0] \qquad \hat{e}^I_k \times \hat{e}^I_k = [1,1] \qquad \hat{e}^I_k / \hat{e}^I_k = [1,1]$$

$$x_k \hat{e}^I_k \pm y_k \hat{e}^I_k = \left(x_k \pm y_k\right)\hat{e}^I_k \qquad \hat{e}^I_k \times \hat{e}^I_j = [-1,+1] \qquad x_k \hat{e}^I_k \times y_k \hat{e}^I_k = x_k y_k \left(\hat{e}^I_k\right)^2 = x_k y_k [1,1]$$

Serviceability analysis with interval uncertainties

Interval loading and structural parameters

$$\mathbf{x}^I = \left\{ \mu_{\omega p}^I \quad \sigma_{\omega p}^I \quad \alpha^I \quad G^I \quad \omega_j^I \quad \xi_j^I \quad M_j^I \right\}^{\mathrm{T}}$$

Interval standard deviation

$$\sigma_{\ddot{p}_j}^I = \frac{\alpha^I G^I}{4 M_j^I} \sqrt{\frac{\pi N_p}{\xi_j^I \sqrt{2\pi}} \frac{\omega_j^I}{\sigma_{\omega p}^I} \exp\left[-\frac{1}{2} \left(\frac{\omega_j^I - \mu_{\omega p}^I}{\sigma_{\omega p}^I} \right)^2 \right]}$$

$$\underline{\sigma}_{\ddot{p}_j} = \min_{\mathbf{x} \in \mathbf{x}^I} \left\{ \sigma_{\ddot{p}_j} \right\} \qquad \overline{\sigma}_{\ddot{p}_j} = \max_{\mathbf{x} \in \mathbf{x}^I} \left\{ \sigma_{\ddot{p}_j} \right\}$$

Interval CDF of the maximum acceleration

$$L_{\ddot{p}_{j,\max}}^I(b) = \exp\left[-2 \frac{NL}{0.9} \frac{\omega_j^I}{\mu_{\omega p}^I} \exp\left(-\frac{b^2}{2\left(\sigma_{\ddot{p}_j}^I\right)^2} \right) \right]$$

$$\underline{L}_{\ddot{p}_{j,\max}}(b) = \min_{\mathbf{x} \in \mathbf{x}^I} \left\{ L_{\ddot{p}_{j,\max}}(b) \right\} \qquad \overline{L}_{\ddot{p}_{j,\max}}(b) = \max_{\mathbf{x} \in \mathbf{x}^I} \left\{ L_{\ddot{p}_{j,\max}}(b) \right\}$$

Interval expected value of the maximum acceleration

$$\ddot{q}_{\max}^I = \left[\sqrt{2 \ln\left(2 \frac{NL}{0.9} \frac{\omega_j^I}{\mu_{\omega p}^I} \right)} + \frac{0.5772}{\sqrt{2 \ln\left(2 \frac{NL}{0.9} \frac{\omega_j^I}{\mu_{\omega p}^I} \right)}} \right] \sigma_{\ddot{p}_j}^I$$

$$\underline{\ddot{q}}_{\max} = \min_{\mathbf{x} \in \mathbf{x}^I} \left\{ \ddot{q}_{\max} \right\} \qquad \overline{\ddot{q}}_{\max} = \max_{\mathbf{x} \in \mathbf{x}^I} \left\{ \ddot{q}_{\max} \right\}$$

Approach 1: Interval comfort class

$$C^I = C^{(i)} \left[a_{\min}^{(i)} \le \ddot{q}_{\max}^I < a_{\max}^{(i)} \right]$$

$$\underline{C} = C^{(i)} \left[a_{\min}^{(i)} \le \overline{\ddot{q}}_{\max} < a_{\max}^{(i)} \right] \qquad \overline{C} = C^{(i)} \left[a_{\min}^{(i)} \le \underline{\ddot{q}}_{\max} < a_{\max}^{(i)} \right]$$

Approach 2: Interval probability of falling at least in a comfort class

$$\Pr^I \left[C^{(i)} \right] = L_{\ddot{p}_{j,\max}}^I (a_{\max}^{(i)})$$

$$\underline{\Pr}\left[C^{(i)} \right] = \underline{L}_{\ddot{p}_{j,\max}} (a_{\max}^{(i)}) \qquad \overline{\Pr}\left[C^{(i)} \right] = \overline{L}_{\ddot{p}_{j,\max}} (a_{\max}^{(i)})$$

Closed-form solution (1/3)

Interval frequency ratio

$$\rho_j^I = \frac{\omega_j^I}{\mu_{\omega p}^I} \qquad \rho_{j,0} = \frac{\omega_{j,0}}{\mu_{\omega p,0}}; \quad \underline{\rho}_j = \frac{\underline{\omega}_j}{\overline{\mu}_{\omega p}}; \quad \overline{\rho}_j = \frac{\overline{\omega}_j}{\underline{\mu}_{\omega p}}$$

Interval coefficient of variation

$$V_{\omega p}^I = \frac{\sigma_{\omega p}^I}{\mu_{\omega p}^I} \qquad V_1 = \frac{\overline{\sigma}_{\omega p}}{\overline{\mu}_{\omega p}}; \quad V_2 = \frac{\underline{\sigma}_{\omega p}}{\overline{\mu}_{\omega p}} \equiv \underline{V}_{\omega p}; \quad V_3 = \frac{\overline{\sigma}_{\omega p}}{\underline{\mu}_{\omega p}} \equiv \overline{V}_{\omega p}; \quad V_4 = \frac{\underline{\sigma}_{\omega p}}{\underline{\mu}_{\omega p}}$$

Interval standard deviation

$$\sigma_{\ddot{p}_j}^I = \frac{\alpha^I G^I}{4 M_j^I} \sqrt{\frac{\pi N_p}{\xi_j^I \sqrt{2\pi}} \frac{\rho_j^I}{V_{\omega p}^I} \exp\left[-\frac{1}{2} \left(\frac{\rho_j^I - 1}{V_{\omega p}^I} \right)^2 \right]}$$

LB

$$\underline{\sigma}_{\ddot{p}_j}^I \equiv \underline{\sigma}_{\ddot{p}_j}\left(\rho_j^I, V_{\omega p}^I\right) = \frac{\underline{\alpha}\,\underline{G}}{4 \overline{M}_j} \sqrt{\frac{\pi N_p}{\overline{\xi}_j \sqrt{2\pi}\, V_{\omega p}^I} \rho_j^I \exp\left[-\frac{1}{2} \left(\frac{\rho_j^I - 1}{V_{\omega p}^I} \right)^2 \right]}$$

UB

$$\tilde{\sigma}_{\ddot{p}_j}^I \equiv \tilde{\sigma}_{\ddot{p}_j}\left(\rho_j^I, V_{\omega p}^I\right) = \frac{\overline{\alpha}\,\overline{G}}{4 \underline{M}_j} \sqrt{\frac{\pi N_p}{\underline{\xi}_j \sqrt{2\pi}\, V_{\omega p}^I} \rho_j^I \exp\left[-\frac{1}{2} \left(\frac{\rho_j^I - 1}{V_{\omega p}^I} \right)^2 \right]}$$

Closed-form solution (2/3)

Interval frequency ratio

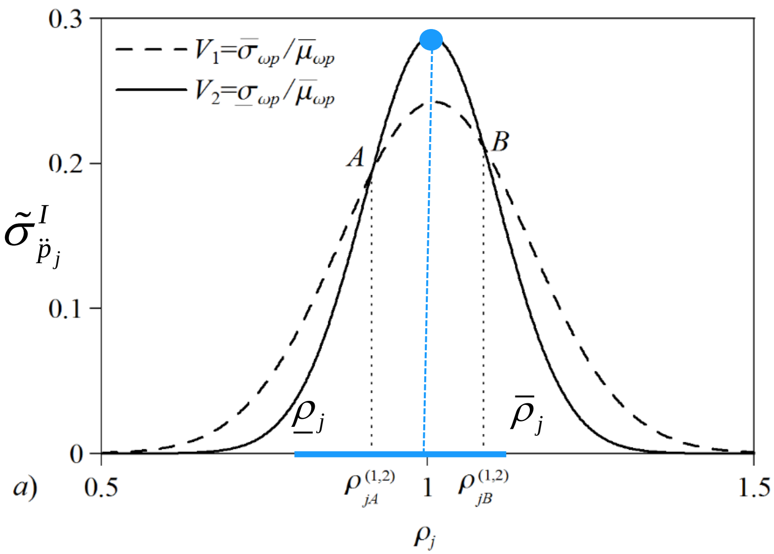
$$\rho_j^I = \frac{\omega_j^I}{\mu_{\omega p}^I} \qquad \rho_{j,0} = \frac{\omega_{j,0}}{\mu_{\omega p,0}}; \quad \underline{\rho}_j = \frac{\underline{\omega}_j}{\overline{\mu}_{\omega p}}; \quad \overline{\rho}_j = \frac{\overline{\omega}_j}{\underline{\mu}_{\omega p}}$$

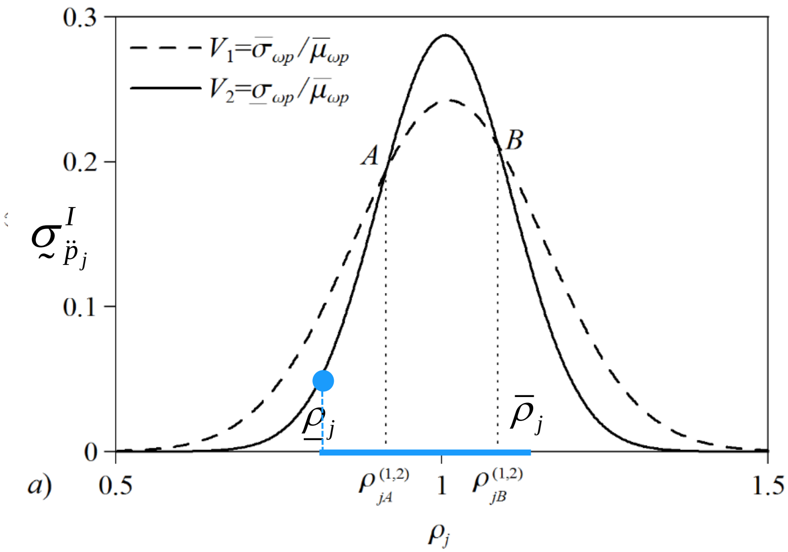
Interval coefficient of variation

$$V_{\omega p}^I = \frac{\sigma_{\omega p}^I}{\mu_{\omega p}^I} \qquad V_1 = \frac{\overline{\sigma}_{\omega p}}{\overline{\mu}_{\omega p}}; \quad V_2 = \frac{\underline{\sigma}_{\omega p}}{\overline{\mu}_{\omega p}} \equiv \underline{V}_{\omega p}; \quad V_3 = \frac{\overline{\sigma}_{\omega p}}{\underline{\mu}_{\omega p}} \equiv \overline{V}_{\omega p}; \quad V_4 = \frac{\underline{\sigma}_{\omega p}}{\underline{\mu}_{\omega p}}$$

Interval standard deviation

$$\sigma_{\ddot{p}_j}^I = \frac{\alpha^I G^I}{4 M_j^I} \sqrt{\frac{\pi N_p}{\xi_j^I \sqrt{2\pi}} \frac{\rho_j^I}{V_{\omega p}^I} \exp\left[-\frac{1}{2} \left(\frac{\rho_j^I - 1}{V_{\omega p}^I} \right)^2 \right]}$$





Intersection points A,B

$$\left. \begin{array}{l} \rho_{jA}^{(i,k)} \\ \rho_{jB}^{(i,k)} \end{array} \right\} = 1 \mp \frac{V_i V_k}{V_i^2 - V_k^2} \sqrt{2(V_i^2 - V_k^2) \log\left(\frac{V_i}{V_k} \right)} \qquad V_i > V_k, \quad (i = 1, \ k=2; \ i = 3, \ k=4)$$

Closed-form solution (2/3)

Interval frequency ratio

$$\rho_j^I = \frac{\omega_j^I}{\mu_{\omega p}^I}$$

$$\rho_{j,0} = \frac{\omega_{j,0}}{\mu_{\omega p,0}}; \quad \underline{\rho}_j = \frac{\underline{\omega}_j}{\overline{\mu}_{\omega p}}; \quad \overline{\rho}_j = \frac{\overline{\omega}_j}{\underline{\mu}_{\omega p}}$$

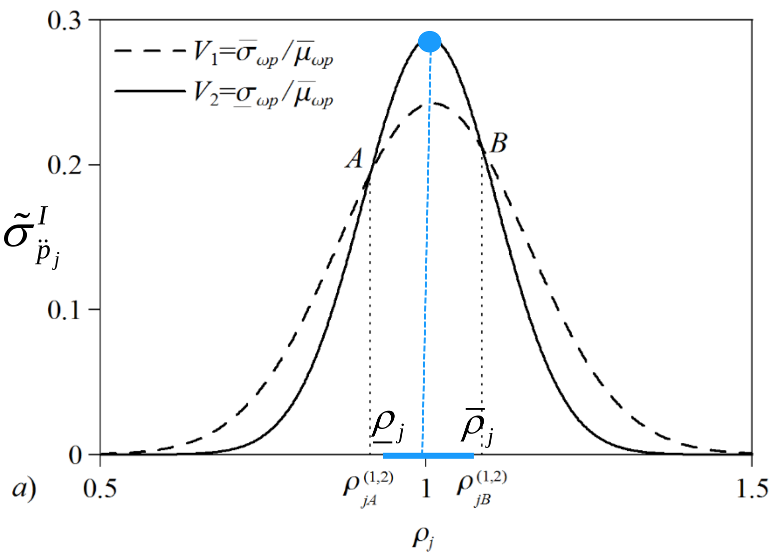
Interval coefficient of variation

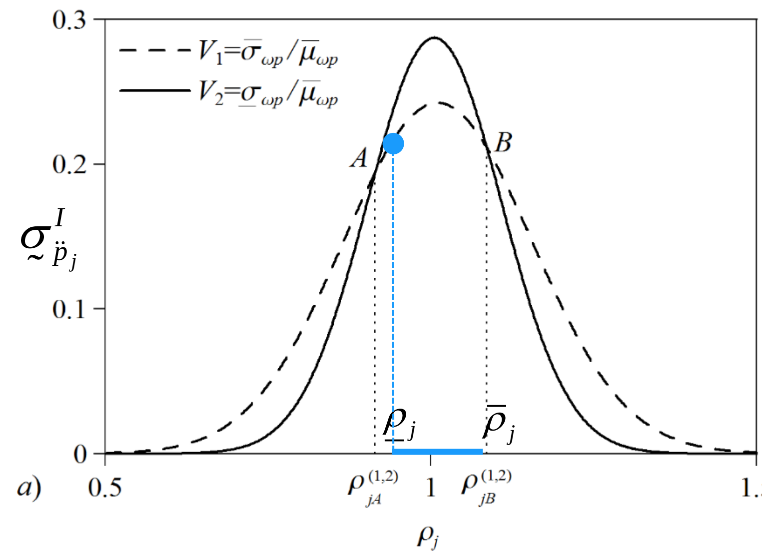
$$V_{\omega p}^I = \frac{\sigma_{\omega p}^I}{\mu_{\omega p}^I}$$

$$V_1 = \frac{\overline{\sigma}_{\omega p}}{\overline{\mu}_{\omega p}}; \quad V_2 = \frac{\underline{\sigma}_{\omega p}}{\overline{\mu}_{\omega p}} \equiv \underline{V}_{\omega p}; \quad V_3 = \frac{\overline{\sigma}_{\omega p}}{\underline{\mu}_{\omega p}} \equiv \overline{V}_{\omega p}; \quad V_4 = \frac{\underline{\sigma}_{\omega p}}{\underline{\mu}_{\omega p}}$$

Interval standard deviation

$$\sigma_{\ddot{p}_j}^I = \frac{\alpha^I G^I}{4M_j^I}\sqrt{\frac{\pi N_p}{\xi_j^I\sqrt{2\pi}}\frac{\rho_j^I}{V_{\omega p}^I}\exp\left[-\frac{1}{2}\left(\frac{\rho_j^I - 1}{V_{\omega p}^I}\right)^2\right]}$$





Intersection points A,B

$$\left.\begin{matrix}\rho_{jA}^{(i,k)} \\ \rho_{jB}^{(i,k)}\end{matrix}\right\} = 1 \mp \frac{V_i V_k}{V_i^2 - V_k^2}\sqrt{2(V_i^2 - V_k^2)\log\left(\frac{V_i}{V_k}\right)} \qquad V_i > V_k, \quad (i=1,\ k=2;\ i=3,\ k=4)$$

Closed-form solution (3/3)

Interval frequency ratio $\quad \rho_j^I = \dfrac{\omega_j^I}{\mu_{\omega p}^I} \qquad \rho_{j,0} = \dfrac{\omega_{j,0}}{\mu_{\omega p,0}}; \quad \underline{\rho}_j = \dfrac{\underline{\omega}_j}{\overline{\mu}_{\omega p}}; \quad \overline{\rho}_j = \dfrac{\overline{\omega}_j}{\underline{\mu}_{\omega p}}$

Interval coefficient of variation $\quad V_{\omega p}^I = \dfrac{\sigma_{\omega p}^I}{\mu_{\omega p}^I} \qquad V_1 = \dfrac{\overline{\sigma}_{\omega p}}{\overline{\mu}_{\omega p}}; \quad V_2 = \dfrac{\underline{\sigma}_{\omega p}}{\overline{\mu}_{\omega p}} \equiv \underline{V}_{\omega p}; \quad V_3 = \dfrac{\overline{\sigma}_{\omega p}}{\underline{\mu}_{\omega p}} \equiv \overline{V}_{\omega p}; \quad V_4 = \dfrac{\underline{\sigma}_{\omega p}}{\underline{\mu}_{\omega p}}$

Intersection points

$$\left.\begin{matrix}\rho_{jA}^{(i,k)} \\ \rho_{jB}^{(i,k)}\end{matrix}\right\} = 1 \mp \frac{V_i V_k}{V_i^2 - V_k^2}\sqrt{2(V_i^2 - V_k^2)\log\left(\frac{V_i}{V_k}\right)} \qquad V_i > V_k, \quad (i=1,\ k=2;\ i=3,\ k=4)$$

Operational Procedure

Estimate $\quad \rho_{j,0};\ \underline{\rho}_j;\ \overline{\rho}_j \qquad V_i\,(i=1,..4) \qquad \rho_{jA}^{(i,k)}, \rho_{jB}^{(i,k)} \quad (i=1,\ k=2;\ i=3,\ k=4)$

$\rho_{j,0} > 1$

$$\underline{\sigma}_{\ddot p_j} = \begin{cases} \underset{\sim}{\sigma}_{\ddot p_j}\left(\overline{\rho}_j, V_3\right) & \text{if} \quad \overline{\rho}_j \le \rho_{jB}^{(3,4)} \\ \underset{\sim}{\sigma}_{\ddot p_j}\left(\overline{\rho}_j, V_4\right) & \text{if} \quad \overline{\rho}_j > \rho_{jB}^{(3,4)} \end{cases}$$

$$\overline{\sigma}_{\ddot p_j} = \begin{cases} \tilde{\sigma}_{\ddot p_j}\left(1, V_2\right) & \text{if} \quad \underline{\rho}_j \le 1 \\ \tilde{\sigma}_{\ddot p_j}\left(\underline{\rho}_j, V_2\right) & \text{if} \quad 1 < \underline{\rho}_j < \rho_{jB}^{(1,2)} \\ \tilde{\sigma}_{\ddot p_j}\left(\underline{\rho}_j, V_1\right) & \text{if} \quad \underline{\rho}_j > \rho_{jB}^{(1,2)} \end{cases}$$

$\rho_{j,0} \le 1$

$$\underline{\sigma}_{\ddot p_j} = \begin{cases} \underset{\sim}{\sigma}_{\ddot p_j}\left(\underline{\rho}_j, V_2\right) & \text{if} \quad \underline{\rho}_j \le \rho_{jA}^{(1,2)} \\ \underset{\sim}{\sigma}_{\ddot p_j}\left(\underline{\rho}_j, V_1\right) & \text{if} \quad \underline{\rho}_j > \rho_{jA}^{(1,2)} \end{cases}$$

$$\overline{\sigma}_{\ddot p_j} = \begin{cases} \tilde{\sigma}_{\ddot p_j}\left(\overline{\rho}_j, V_3\right) & \text{if} \quad \overline{\rho}_j < \rho_{jA}^{(3,4)} \\ \tilde{\sigma}_{\ddot p_j}\left(\overline{\rho}_j, V_4\right) & \text{if} \quad \rho_{jA}^{(3,4)} < \overline{\rho}_j < 1 \\ \tilde{\sigma}_{\ddot p_j}\left(1, V_2\right) & \text{if} \quad \overline{\rho}_j \ge 1 \end{cases}$$

Numerical application (1/2) Approach 1

Interval loading and structural parameters

Ideal Footbridge

L=50 m b=2 m N_p = 50

Setra $\ddot{q}_{\max}^{(S)} = 2.15 \text{ m/s}^2$

Comfort class $C^{(3)}$

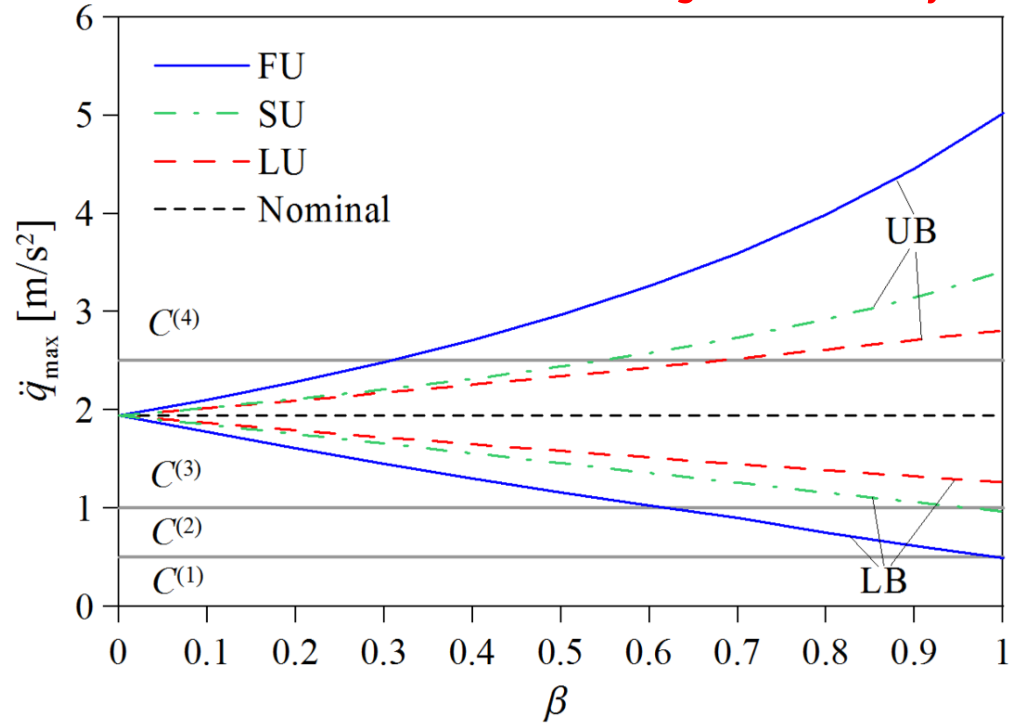
$$\mathbf{x}^I = \left\{ \mu_{\omega p}^I \quad \sigma_{\omega p}^I \quad \alpha^I \quad G^I \quad \omega_j^I \quad \xi_j^I \quad M_j^I \right\}^T$$

FU: Full Uncertainty

SU: Structural Uncertainty

LU: Loading Uncertainty

Parameter	$x_{0,k}$	$\Delta\chi_k$
$\mu_{\omega p}$ [rad/s]	2 π 1.88	0.03
$\sigma_{\omega p}$ [rad/s]	2 π 0.17	0.17
α [ ]	0.35	0.20
G [N]	700	0.10
ω_j [rad/s]	2 π 1.88	0.10
ξ_j [%]	0.42	0.60
M_j [kg]	5000	0.10



$$x_k^I(\beta) = x_{k,0}\left(1 + \beta \Delta\chi_k \hat{e}_k^I\right) \qquad 0 \le \beta \le 1$$

Numerical application (2/2) Approach 2

Interval loading and structural parameters
Ideal Footbridge

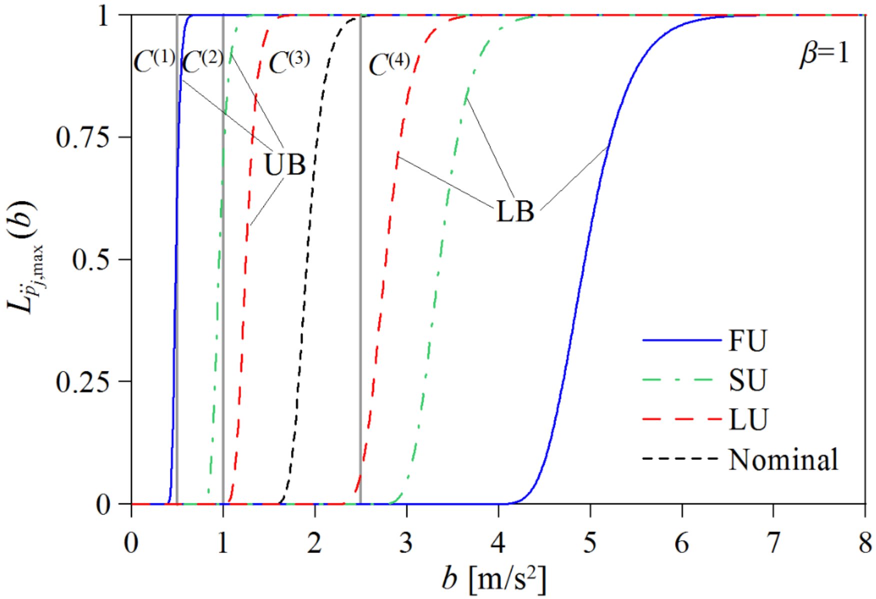
L=50 m b=2 m N_p = 50

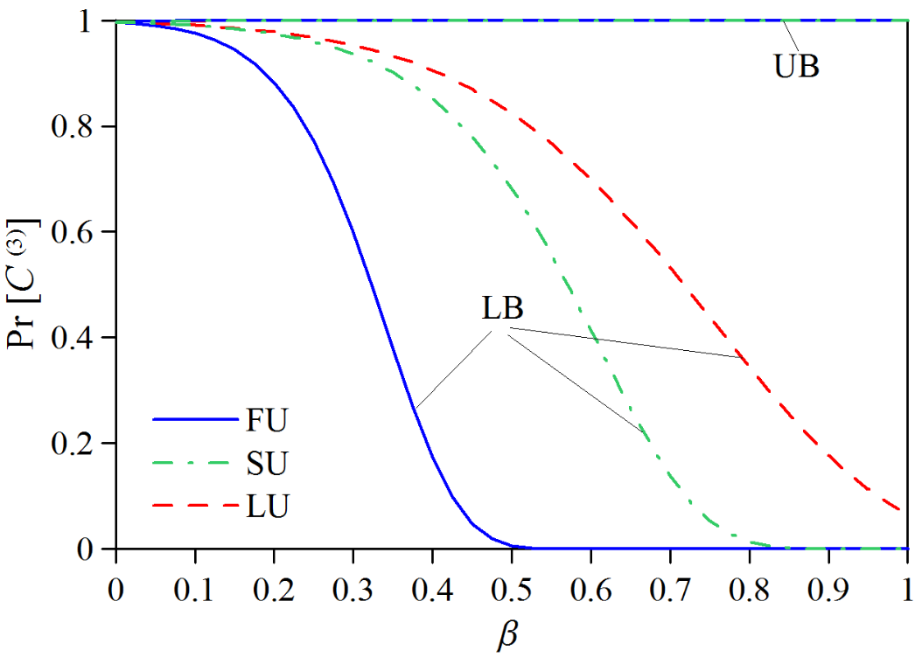
$$\mathbf{x}^I = \left\{ \mu^I_{\omega p} \quad \sigma^I_{\omega p} \quad \alpha^I \quad G^I \quad \omega^I_j \quad \xi^I_j \quad M^I_j \right\}^T$$

FU: Full Uncertainty

SU: Structural Uncertainty

LU: Loading Uncertainty





$$x^I_k(\beta) = x_{k,0}\left(1 + \beta \Delta\chi_k \hat{e}^I_k\right) \qquad 0 \le \beta \le 1$$

Conclusions

- Serviceability assessment of footbridges is studied through a non-deterministic approach, modeling structural and loading parameters as interval variables.
- Two approaches are introduced for the comfort evaluation.
- An analytical procedure is formulated, which allows deriving accurate closed-form estimates of the bounds of the quantities of interest.
- The high degree of uncertainty in the loading and structural parameters leads to very large intervals of variation of the response.

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