9TH INTERNATIONAL WORKSHOP ON RELIABLE ENGINEERING COMPUTING
Risk and Uncertainty in Engineering Computations
Virtual Conference: May 17-20, 2021
REC2021

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Uncertainty Analysis of Fatigue Failure Using an Interval Approach

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Introduction

- The objective of this research is to develop a methodolgy for predicting the remaining fatigue life of a structure considering uncertainties
- The research motivation is to handle the presence of uncertainties in both laboratory and field data when limmited information is available
- The procedure uses a possibilistic method via the application of interval variables to enumerate uncertainties

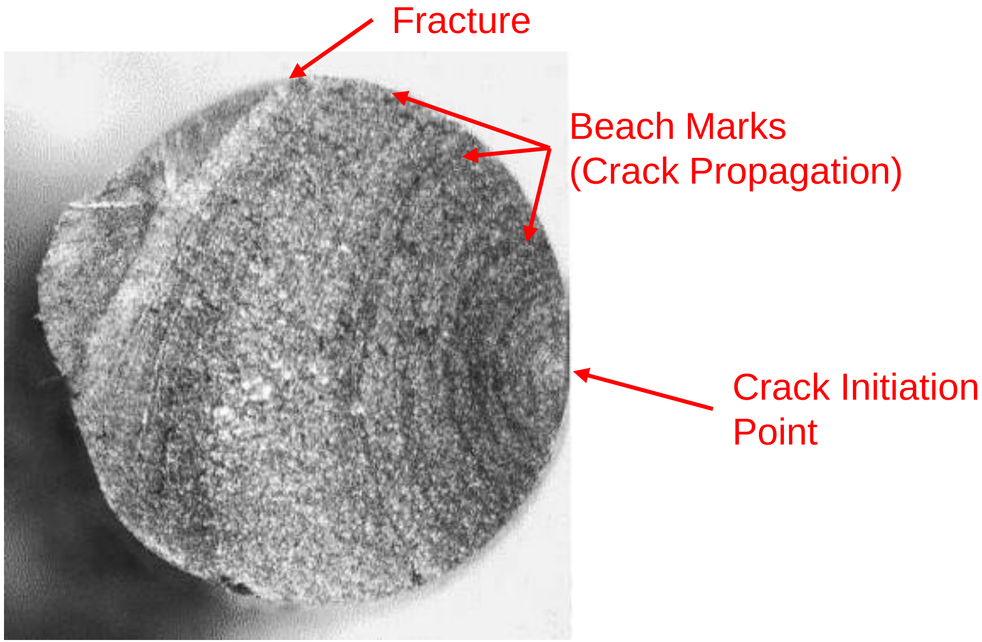
Fatigue

- Structures, subjected to cyclic loads, may experience material fatigue failure
- Occurs when the micro-structural damage caused at each cycle of load is significantly accumulated
- Failure can occur even though the stresses induced by the applied cyclic loads are within allowable design criteria
- This makes fatigue failure an important design concern for any structure subject to cyclic loading

Fatigue

- Fatigue starts with a *crack initiation* phase
  - Atomic-scale defects in the material are subject to concentrated stresses, nucleate *many* micro-cracks
  - The development of these micro-cracks makes up the bulk of the fatigue life of a component
- Following crack initiation is the *crack propagation* phase
  - A dominant crack emerges (by now, typically large enough to be seen with the naked eye)
  - The crack elongates causing significant cross sectional area loss until rupture occurs

Fatigue


Fracture
Beach Marks
(Crack Propagation)
Crack Initiation
Point

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Fatigue

Fatigue induced crack on the northbound Lake Shore Drive approach to Outer Drive Bridge Chicago, IL, Feb 2019

Fatigue Life Prediction

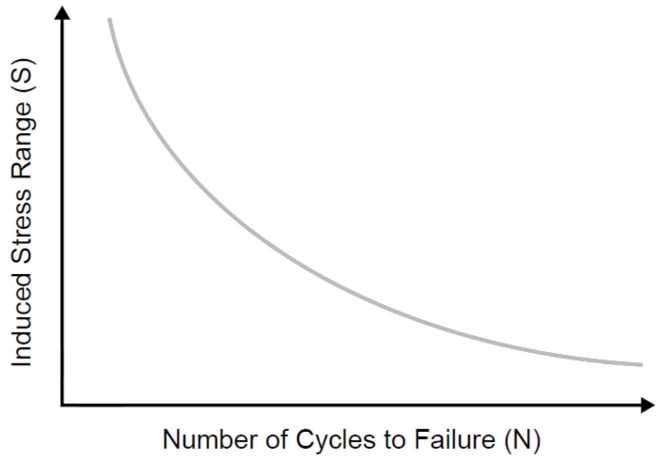
- The relationship between the cyclic strain of a structural component and its cycles to failure is obtained through laboratory test data acquisition
- This relationship can be rearranged to represent the relationship between cycles to failure (*N*) and stress range (*S*) (Basquin, 1910)

$$N = \frac{C}{S^m}$$

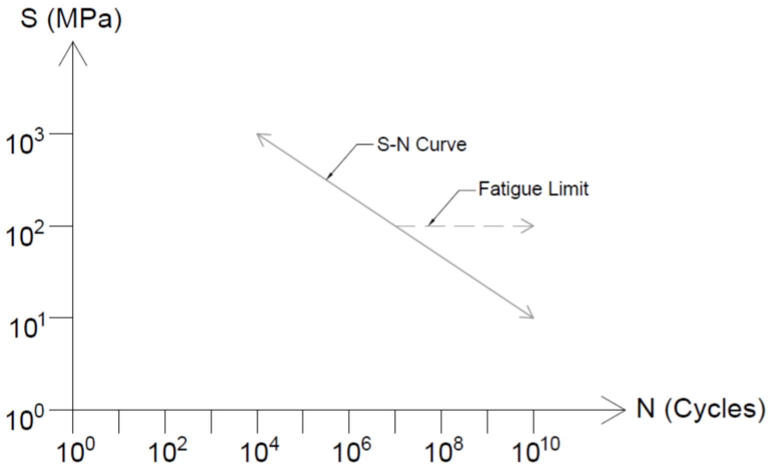
- *C* is the fatigue coefficient, and *m* is the fatigue exponent
- The coefficient and exponent are determined experimentally

Fatigue Life Prediction

A typical S-N relation (S-N curve)



S-N curve for base-metal steel



Fatigue Life Prediction

- The S-N relationship is defined for a single stress range, where the stress varies between a maximum stress and minimum stress
- However, components in service can be subject to many stress ranges over the course of their service lives
- A cumulative damage model can be used to consider the effects of multiple stress ranges
  - Each cycle causes a small amount of damage to the system
  - Over the course of the system's service life, the damage accrued until it reaches a critical value that causes failure

Fatigue Life Prediction

- Miner (1945) defined a simple cumulative damage model

$$D = \sum \frac{n_i}{N_i}$$

- D is the total damage, n_i is the number of cycles undergone at stress range i, and N_i is the number of cycles to failure at stress range i
- Failure generally occurs when the total damage equals 1
- Combining Miner's rule and the S-N relation:

$$D = \frac{1}{C} \cdot \sum_i n_i \cdot S_i^m$$

Remaining Fatigue Life Prediction

Remaining life prediction follows these steps:

- Identify a mean damage accumulation rate D_m
    - D_d is the damage calculated (D)
    - t_d is the time over which the damage was calculated
- Calculate the total fatigue lifetime t_l
- Calculate the remaining fatigue lifetime t_r
    - t_a is the age of the structure/component
- The existing damage D_e can also be calculated

$$D_m = \frac{D_d}{t_d}$$

$$t_l = \frac{1}{D_m}$$

$$t_r = t_l - t_a$$

$$D_e = D_m \cdot t_a$$

Uncertainties Present in Fatigue Life Prediction

- Uncertainties arise from both laboratory experiments and field data acquisition. These uncertainties may be present in:
  a) The geometric configuration of the detail
  b) The material composition and behavior of the detail
  c) The stress range measurements
- Uncertainties may also arise from the numerical procedures used to process and analyze the data, including:
  a) Numerical and regression errors from calculating the S-N relationship
  b) Numerical error and the loss of precision when performing cycle counting

Considering Uncertainties through Probability

- Some approaches have used probability distributions to incorporate uncertainties (Ang and Munse 1975; Sen 2006; Kwon et at. 2012; Diekfuss and Foley 2016)
  - In these approaches Insufficient data makes the cumulative damage extremely sensitive to uncertainties

Considering Uncertainties through Possibility

- One approach to alleviate the issue of insufficient data is to use possibility theory via interval variables
- Interval variables have been used in several fatigue problems, such as:
  - Bootstrapping a non-uniform interval S-N curve (Gu and Ma 2018)
  - Determining bounds in the crack propagation problem (Long et al. 2018)
  - Performing fatigue analysis on structures with interval axial stiffness (Sofi et al. 2019)

Methodology

- An interval S-N relation is developed to enumerate the uncertain response to a class of detail made of a specific material
- Stress and cycle count data is obtained for a specific structure to be analyzed and uncertainties are enumerated
- These data sources are then used to predict the existing damage of the structure and predict the remaining fatigue life before failure

Methodology - Step 1

Construct an interval S-N relationship from laboratory test data

- The uncertainty in the laboratory test data is quantified by defining interval fatigue coefficient ***C***
- C is the fatigue coefficient, and δ_{Cl} and δ_{Cu} are the lower and upper variation parameters, respectively. These variation parameters are obtained using an enveloping procedure
- e_j is the set of residual values obtained when regressing C and m

$$\boldsymbol{C} = [\underline{C}, \overline{C}] = C \cdot [\delta_{C_l}, \delta_{C_u}]$$

$$\delta_{C_l} = \exp(\min(e_j))$$

$$\delta_{C_u} = \exp(\max(e_j))$$

Methodology - Step 2

Determine interval stress range and cycle counts from field data

- Interval stress ranges $\boldsymbol{S_i}$ are constructed
- δ_{Sil}, and δ_{Siu} are the lower and upper variation parameters for each stress range, respectively

$$\boldsymbol{S_i} = \left[\underline{S_i}, \overline{S_i}\right] = \left[S_i - \delta_{S_{il}}, S_i + \delta_{S_{iu}}\right]$$

- Values for the parameters can be determined by:
    a) Precision of the sensors
    b) Degree of rounding used in cycle counting
    c) Expert opinion

Methodology - Step 3

Calculate the interval damage accumulated over the duration of the field data

- The interval damage $\boldsymbol{D_d}$ is calculated using:
  - parameters m and $\boldsymbol{C}$ (Step 1)
  - the sets $\boldsymbol{S}_i$ and n_i (Step 2)

$$\boldsymbol{D_d} = \frac{1}{\boldsymbol{C}} \cdot \sum_i n_i \, . \, \boldsymbol{S_i}^m = \frac{1}{[\underline{C}, \overline{C}]} \sum_i n_i \, . \left[\underline{S_i}, \overline{S_i}\right]^m$$

Methodology - Step 4

Calculate the interval existing damage and interval remaining life

- The interval mean damage per unit time $\boldsymbol{D_m}$ is calculated
  - t_d is the duration of the field data collection

$$\boldsymbol{D_m} = \frac{\boldsymbol{D_d}}{t_d} = \left[\frac{\underline{D_d}}{\overline{t_d}}, \frac{\overline{D_d}}{\underline{t_d}}\right]$$

- The interval existing damage $\boldsymbol{D_e}$ is calculated
  - t_a is the current age of the structure

$$\boldsymbol{D_e} = \boldsymbol{D_m} \cdot t_a = \left[\underline{D_m} \cdot t_a, \overline{D_m} \cdot t_a\right]$$

- The interval fatigue life of the structure $\boldsymbol{t_l}$ is calculated

$$\boldsymbol{t_l} = \frac{1}{\boldsymbol{D_m}} = \left[\frac{1}{\overline{D_m}}, \frac{1}{\underline{D_m}}\right]$$

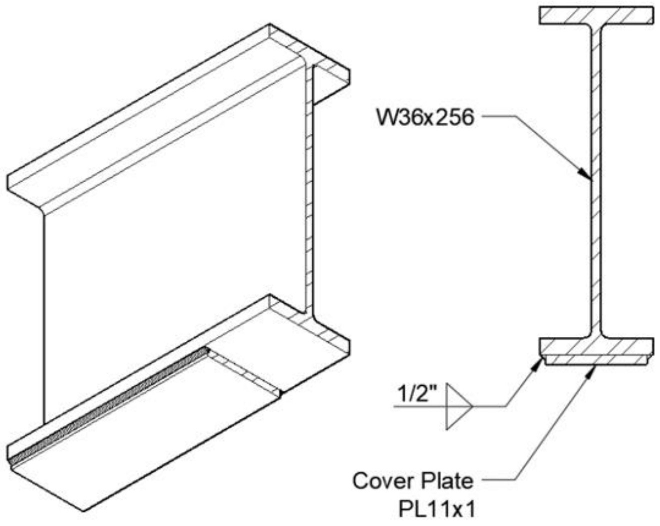
- The interval remaining life $\boldsymbol{t_r}$ is calculated

$$\boldsymbol{t_r} = \boldsymbol{t_l} - t_a = \left[\underline{t_l} - t_a, \overline{t_l} - t_a\right]$$

Numerical Example - Problem Definition

A hypothetical steel girder bridge is analized

- The age of the bridge is 20 years old
- The most fatigue-prone details on the bridge are cover plates on the bottom flanges of the deck girders.
- The cover plate has no transverse welds
- The girder material is A514 steel



Laboratory Test Data

Laboratory test data for this detail/material combination is available from Fisher et al. (1969)

TABLE 1. Laboratory S-N Data (adopted from Fisher et al. 1969)

S_i MPa	(ksi)	$N\ (cycles \times 10^3)$ $(Multiple\ tests)$
55.16	(8)	1988.9, 2916.2, 3409.2
82.74	(12)	1031.1, 848.3, 1310.9, 821.7, 1004.7, 1220.0, 755.2
110.32	(16)	514.8, 1227.8, 854.9, 428.5, 542.2, 598.5, 492.9, 412.5, 589.6, 578.0
137.90	(20)	341.3, 429.1, 445.9, 282.3, 192.3, 339.5, 260.0, 238.8, 374.0, 296.0, 207.0
165.47	(24)	156.6, 213.8, 285.2, 192.5

Field Test Data

- The stress range and cycle count measurements for bridge #0160335 are used for the field test data (adopted from Hahin et al. 1993)
- The cycle counts were obtained over a 24-hour collection period
- It is assumed that for each stress range, there is ±1% uncertainty due to sensor precision

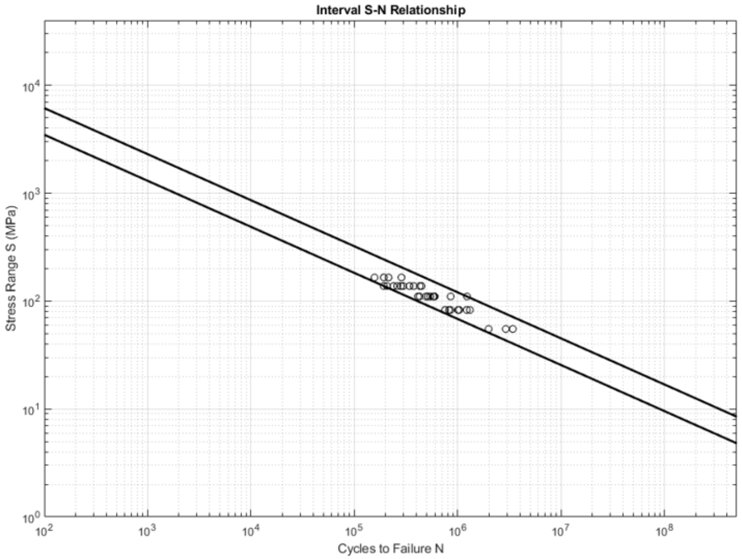
TABLE 2. Stress range and cycle count data (Hahin et al. 1993)

S_i MPa	(ksi)	n_i $(cycles)$
6.89	(1.0)	2843
10.34	(1.5)	991
13.79	(2.0)	386
17.24	(2.5)	111
20.68	(3.0)	88
24.13	(3.5)	64
27.58	(4.0)	33
31.03	(4.5)	32
34.47	(5.0)	15
37.92	(5.5)	7

Problem Solution

Step 1. The laboratory data is regressed and used to determine interval fatigue coefficient ***C*** and fatigue exponent *m*

TABLE 3. Interval fatigue coefficient and fatigue exponent		
Interval Fatigue Coefficient ($\mathbf{C}$) $MPa^m \times 10^{10}$	($ksi^m \times 10^8$)	*Fatigue Exponent (m)*
[1.975,7.478]	[2.145,8.122]	2.342



Problem Solution

Step 2. The interval stress ranges S_i are determined

TABLE 4. Interval stress ranges

Interval Stress Range (S_i) *MPa*	*(ksi)*
[6.83,6.96]	[0.99,1.01]
[10.24,10.45]	[1.49,1.52]
[13.65,13.93]	[1.98,2.02]
[17.06,17.41]	[2.48,2.53]
[20.48,20.89]	[2.97,3.03]
[23.89,24.37]	[3.47,3.54]
[27.30,27.85]	[3.96,4.04]
[30.72,31.34]	[4.46,4.55]
[34.13,34.82]	[4.95,5.05]
[37.54,38.30]	[5.45,5.56]

Problem Solution

Step 3. The interval damage occurring during the one day data collection period:

$$(t_d=1/365 \text{ years})$$

is calculated as:

$$\boldsymbol{D_d}=[1.640,6.504]\times10^{-5}$$

Problem Solution & Verification

Step 4. The values for interval mean damage, interval existing damage (t_a=20 years), interval fatigue life, and interval remaining life are determined.

For verification, a Monte Carlo simulation was conducted using 10^8 realizations of uniformly distributed random variables

TABLE 5. Fatigue damage and life prediction results

Parameter	*Interval Results*	*MCS Results*
Interval mean damage ($\boldsymbol{D_m}$)	[0.0060, 0.0237]	[0.0060,0.0236]
Interval existing damage ($\boldsymbol{D_e}$)	[0.1197, 0.4748]	[0.1204,0.4717]
Interval fatigue life ($\boldsymbol{t_l}$) (years)	[42.12, 167.11]	[42.40,166.08]
Interval remaining life ($\boldsymbol{t_r}$) (years)	[22.12, 147.11]	[22.40,146.08]

Summary

The developed method:

- Considers uncertainties present in both laboratory and field data as well as uncertainties introduced by the analysis process
- Computes sharp bounds on the existing damage and remaining fatigue life of the structure with minimal computation effort
- Interval results and their sharpness obtained by this method are verified by Monte Carlo simulation

Conclusions

- The numerical example demonstrates the hypersensitivity of fatigue analysis to the presence of uncertainties
- These uncertainties may result in large variations in the fatigue life prediction
- It is of utmost importance to minimize the errors and uncertainties in the input of the analysis process to obtain tighter bounds
- The method's simplicity and versatility make it an attractive option to consider uncertainties in the remaining fatigue life problem, *especially when limited data is available*

Thank you for your time

Questions?