Uncertainty Analysis of Fatigue Failure Using an Interval Approach

Michael Desch[1*] and Mehdi Modares[1]

[1]*Department of Civil, Architectural and Environmental Engineering, Illinois Institute of Technology, USA,*
[*]*Corresponding author: mdesch@hawk.iit.edu*

Abstract. Structural and mechanical systems are susceptible to fracture under cyclic loading, which may lead to fatigue failure. Fatigue failure constitutes a multi-phase process starting with a crack initiation phase and continuing into a crack growth phase, which may result in system failure. In the conventional crack initiation method, based on Miner's rule, each cycle of load is considered to cause an infinitesimal amount of damage in the material leading to a fatigue failure (Miner 1945). This approach uses the results of experiments on the relationship between the applied stress and number of cycles (S-N curves) for the determination of the fatigue life for a specific stress range. These deterministic curves are developed for different structural components and materials through data regression (Basquin 1910).

As such, the uncertainties in material properties, geometry and applied loads are not considered. In some applications, probabilistic methods are utilized by incorporating uncertainties using probability distributions for the parameters governing the formula of the S-N curve. However, the cumulative damage is highly sensitive to these uncertainties, which is an inherent characteristic of using the traditional probabilistic approaches.

In this work, a new method for crack initiation prediction is developed through quantification of uncertainties using an interval approach. Using this method, the values of stress ranges as well as fatigue parameters are quantified as interval variables. Then, the existing interval damage is calculated, leading to the determination of upper and lower bounds of the remaining fatigue life. A numerical example illustrating the developed method is presented; and the results are discussed.

Keywords: Fatigue, Damage, Uncertainty, Interval

1. Introduction

Structures, subjected to cyclic loads, may experience material fatigue failure. Material fatigue failure in components of a structure occurs when the damage caused at each cycle of load is significantly accumulated. The failure can occur even though the stresses induced by the applied cyclic loads are within allowable design criteria. Fatigue failure is a multi-phase process that includes a crack initiation phase and a subsequent crack propagation phase. There exist numerous analytical schemes for prediction of both crack initiation and crack propagation phases.

The conventional crack initiation method, based on Miner's rule, follows the material behavior phenomenon that each cycle of load causes an infinitesimal amount of damage in the material. The damage accumulated over numerous cycles leads to a fatigue failure (Miner, 1945). The crack initiation method can also predict the structure's fatigue life. This method uses experimental results relating the applied stress and number of cycles to failure (S-N relationship) for the determination of the fatigue life (Basquin, 1910).

However, in the crack initiation method, the uncertainties in material properties, geometry and applied loads are not considered. Moreover, this method does not consider the inherent variability of the experiments. Although, in some approaches, the uncertainties are incorporated using probability distributions for the parameters governing the formula of the S-N relationship (Ang and Munse 1975; Sen 2006; Kwon et at. 2012; Diekfuss and Foley 2016), a lack of sufficient data makes the cumulative damage highly sensitive to those uncertainties. One of the approaches to alleviate this issue and enumerate the aforementioned variability and uncertainties with insufficient data is to use interval variables. Using interval variables, the uncertainty is enumerated by lower and upper bounds with no assumption of the possible distributions. Interval variables have been used in several fatigue problems, such as bootstrapping a non-uniform interval S-N curve (Gu and Ma 2018), determining bounds in the crack propagation problem (Long et al. 2018), and performing fatigue analysis on structures with interval axial stiffness (Sofi et al. 2019).

In this work, a new method is developed that is capable of predicting the remaining fatigue life of a structure. This method considers and quantifies variability and uncertainties using an interval approach. Using this method, the uncertainties in the values of the fatigue parameters (obtained from laboratory test data) and stress ranges (obtained from field test data) are quantified as interval variables. Following that, the existing interval damage is calculated, leading to the determination of upper and lower bounds of the structure's remaining fatigue life.

2. Background

This work is the symbiosis of two historically independent fields, structural fatigue life prediction, and interval analysis. To represent the background for this work, reviews of both fields are presented.

2.1. Fatigue Life Prediction

The relationship between the cyclic strain of a structural component and its cycles to failure is obtained through laboratory test data acquisition. This relationship can be rearranged to represent the relationship between cycles to failure and stress range as (Basquin, 1910):

$$N = C/S^m \tag{1}$$

where, C is the fatigue coefficient, and m is the fatigue exponent. The coefficient and exponent are determined experimentally. N is the number of cycles until failure when the system is subject to stress range S. Tests of the S-N relationship usually exhibit considerable scatter. Data regression is generally used to calculate values for the parameters C and m. Eq. 1 is defined for a single stress range, where the stress varies between a maximum stress and minimum stress. However, components in service can be subject to many stress ranges over the course of their service lives. To consider the effects of multiple stress ranges, a cumulative damage approach can be used. In such an approach, each cycle causes a small amount of damage to the system. Over the course of the system's service life, the damage accrued until it reaches a critical value that causes failure. A cumulative damage rule is defined as (Miner 1945):

$$D = \Sigma n_i/N_i \tag{2}$$

where D is the total damage, n_i is the number of cycles undergone at stress range i, and N_i is the number of cycles to failure at stress range i. In Miner's model, failure generally occurs when the total damage equals 1. The damage of a structure, subjected to multiple stress ranges, can be calculated by combining Eqs. 1 and 2 as:

$$D = \frac{1}{C} \cdot \sum_i n_i \cdot S_i^m \tag{3}$$

When a structure is subject to a combination of different stress ranges, a cycle counting method may be used to analyze the stress data. The method will produce an equivalent number of cycles corresponding to a finite number of stress ranges. One of the most common cycle-counting methods used is called the rain-flow method (Matsuishi and Endo 1968).

2.2. S-N Data Regression

Fatigue parameters C and m are regressed from S-N data by first transforming both variables S and N to a logarithmic space as $\ln S$ and $\ln N$. In this transformed space, the S-N data forms a linear relationship, and the parameters are determined by least squares regression as:

$$C = \exp\left(\frac{(\Sigma \ln N)(\Sigma (\ln S)^2) - (\Sigma \ln S)(\Sigma \ln S \ln N)}{p(\Sigma (\ln S)^2) - (\Sigma \ln S)^2}\right) \tag{4}$$

$$m = -\frac{p(\Sigma \ln S \ln N) - (\Sigma \ln S)(\Sigma \ln N)}{p(\Sigma (\ln S)^2) - (\Sigma \ln S)^2} \tag{5}$$

where p is the number of data points. In the least squared procedure, the residual value of each data point e_j is calculated as:

$$e_j = \ln N_j - \ln C + m \ln S_j \tag{6}$$

2.3. Uncertainties Present in Fatigue Life Prediction

The equations as presented in section 2.1 determine the structure's fatigue behavior deterministically. However, the uncertainties present in the fatigue life prediction problem must be considered in order to characterize the variations in the fatigue life prediction.

Uncertainties arise from both laboratory experiments and field data acquisition. These uncertainties may be present in 1) the geometric configuration of the detail, 2) the material composition and behavior of the detail, and/or 3) the stress range measurements. Uncertainties may also arise from the numerical procedures used to process and analyze the data. These include: 1) numerical and regression errors from calculating the S-N relationship, and 2) numerical error and the loss of precision when performing cycle counting.

The parameters affected by these uncertainties are the stress ranges S_i and the fatigue parameters (C and m). As the uncertainty in the fatigue parameters are perfectly correlated (Ang and Munse 1975), all uncertainty associated with these parameters may be described as uncertainty in a single parameter. Therefore, the uncertainty in the fatigue parameters may be described with C whilst m remains deterministic.

2.4. INTERVAL ANALYSIS

An interval is defined as a real, closed set bounded by upper and lower values as:

$$\boldsymbol{X} = [\underline{x}, \overline{x}] = \{x \in \mathbb{R} \mid \underline{x} \leq x \leq \overline{x}\} \tag{7}$$

in which $\underline{x}$ is the lower bound and $\overline{x}$ is the upper bound of the interval. In this paper, interval variables are depicted in bold. Interval analysis describes the mathematical methods used to define, operate on, and compare interval sets. The set resulting from an interval operation contains all possible values of that operation between all members of the original interval set(s). Interval Analysis and the definitions for its mathematical methods were introduced by Moore (1966). The basic binary operators are amongst those definitions.

Additionally, there has been recent work to fully define the interval power operation; Heimlich (2013). If positive interval $\boldsymbol{X} = \{x \in \mathbb{R}^+ \mid \underline{x} \leq x \leq \overline{x}\}$ is raised to a deterministic power y, the result is evaluated as:

$$\boldsymbol{X}^y = [\underline{x}, \overline{x}]^y = [\min(\underline{x}^y, \overline{x}^y), \max(\underline{x}^y, \overline{x}^y)] \tag{8}$$

3. Methodology

3.1. FORMULATION OF INTERVAL FATIGUE FAILURE ANALYSIS

The general algorithm and major steps for interval fatigue failure analysis and life prediction are given below.

Step 1. Construct an interval S-N relationship from laboratory test data
The uncertainty in the laboratory test data is quantified by defining interval fatigue coefficient $\boldsymbol{C}$ as:

$$\boldsymbol{C} = [\underline{C}, \overline{C}] = C \cdot [\delta_{C_l}, \delta_{C_u}] \tag{9}$$

where, C is the fatigue coefficient, and δ_{C_l} and δ_{C_u} are the lower and upper variation parameters, respectively. These variation parameters are obtained using an enveloping procedure as:

$$\delta_{C_l} = \exp(\min(e_j)) \tag{10}$$

$$\delta_{C_u} = \exp(\max(e_j)) \tag{11}$$

The interval S-N relationship is constructed using interval fatigue coefficient $\boldsymbol{C}$ and fatigue exponent m.

Step 2. Determine interval stress range and cycle counts from field data
Interval stress ranges $\boldsymbol{S_i}$ are constructed as:

$$\boldsymbol{S_i} = \left[\underline{S_i}, \overline{S_i}\right] = \left[S_i - \delta_{S_{i_l}}, S_i + \delta_{S_{i_u}}\right] \tag{12}$$

where, $\delta_{S_{i_l}}$, and $\delta_{S_{i_u}}$ are the lower and upper variation parameters for each stress range, respectively. Values for the variation parameters can be determined by a) the precision of the sensors used to collect the field data, b) the degree of rounding used in the cycle counting method, and/or c) expert opinion.

Step 3. Calculate the interval damage accumulated over the duration of the field data
The interval damage ($\boldsymbol{D_d}$) that is accumulated over the duration of the field data collection is calculated using parameters m and $\boldsymbol{C}$ (Step 1) and the sets $\boldsymbol{S_i}$ and n_i (Step 2) as:

$$\boldsymbol{D_d} = \frac{1}{\boldsymbol{C}} \cdot \sum_i n_i \,.\, \boldsymbol{S_i}^m = \frac{1}{\left[\underline{C}, \overline{C}\right]} \sum_i n_i \,.\, \left[\underline{S_i}, \overline{S_i}\right]^m \tag{13}$$

Step 4. Calculate the interval existing damage and interval remaining life
The interval mean damage per unit time $\boldsymbol{D_m}$ is calculated as:

$$\boldsymbol{D_m} = \frac{\boldsymbol{D_d}}{t_d} = \left[\frac{\underline{D_d}}{t_d}, \frac{\overline{D_d}}{t_d}\right] \tag{14}$$

where, t_d is the duration of the field data collection. In terms of $\boldsymbol{D_m}$, the interval existing damage $\boldsymbol{D_e}$ is:

$$\boldsymbol{D_e} = \boldsymbol{D_m} \cdot t_a = \left[\underline{D_m} \cdot t_a, \overline{D_m} \cdot t_a\right] \tag{15}$$

where, t_a is the current age of the structure. Then, the interval fatigue life of the structure $\boldsymbol{t_l}$ is:

$$\boldsymbol{t_l} = \frac{1}{\boldsymbol{D_m}} = \left[\frac{1}{\overline{D_m}}, \frac{1}{\underline{D_m}}\right] \tag{16}$$

Finally, the interval remaining life $\boldsymbol{t_r}$ is calculated as:

$$\boldsymbol{t_r} = \boldsymbol{t_l} - t_a = \left[\underline{t_l} - t_a, \overline{t_l} - t_a\right] \tag{17}$$

The lower bound of $\boldsymbol{t_r}$, that is $\underline{t_r}$, can then be conservatively taken as the structure's minimum remaining fatigue life.

4. Numerical Example

4.1. Problem Definition

In this example problem, an uncertainty analysis using the developed method is performed to obtain the interval existing damage and interval remaining life for a bridge. To verify the obtained interval bounds, a Monte Carlo simulation (MCS) is also conducted.

The bridge was constructed 20 years ago. The steel grade used was A514. The most fatigue prone details on the bridge are identified as the cover plates on the bottom flanges of the deck girders (Figure 1).

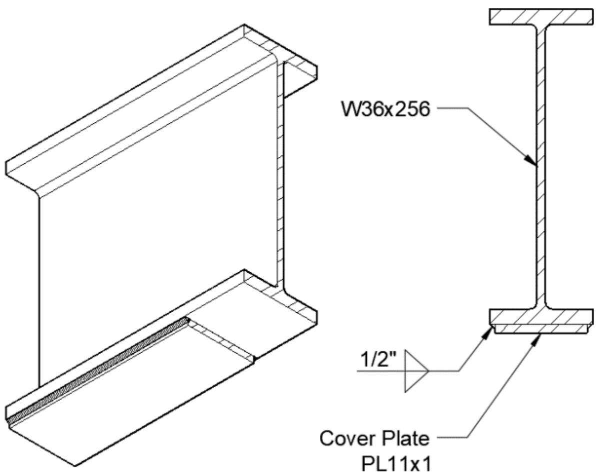

Figure 1. *Cover plate detail, ends un-welded*

The cover plates have un-welded ends. Laboratory test data for this detail/material combination is available from Fisher et al. (1969) (Table I).

Table I. Laboratory S-N Data (adopted from Fisher et al. 1969)

S_i MPa	(ksi)	N $(cycles \times 10^3)$ $(Multiple\ tests)$
55.16	(8)	1988.9, 2916.2, 3409.2
82.74	(12)	1031.1, 848.3, 1310.9, 821.7, 1004.7, 1220.0, 755.2
110.32	(16)	514.8, 1227.8, 854.9, 428.5, 542.2, 598.5, 492.9, 412.5, 589.6, 578.0
137.90	(20)	341.3, 429.1, 445.9, 282.3, 192.3, 339.5, 260.0, 238.8, 374.0, 296.0, 207.0
165.47	(24)	156.6, 213.8, 285.2, 192.5

The stress range and cycle count measurements for bridge #0160335 are used for the field test data (adopted from Hahin et al. 1993) (Table II). The cycle counts were obtained over a 24-hour collection period. It is assumed that for each stress range, there is $\pm 1\%$ uncertainty due to sensor precision.

Table II. Stress range and cycle count data (Hahin et al. 1993)

S_i MPa	(ksi)	n_i $(cycles)$
6.89	(1.0)	2843
10.34	(1.5)	991
13.79	(2.0)	386
17.24	(2.5)	111
20.68	(3.0)	88
24.13	(3.5)	64
27.58	(4.0)	33
31.03	(4.5)	32
34.47	(5.0)	15
37.92	(5.5)	7

4.2. PROBLEM SOLUTION

Step 1. The laboratory data is regressed and used to determine interval fatigue coefficient $\boldsymbol{C}$ and fatigue exponent m (Table III).

Table III. Interval fatigue coefficient and fatigue exponent

Interval Fatigue Coefficient $(\boldsymbol{C})$ $MPa^m \times 10^{10}$	$(ksi^m \times 10^8)$	*Fatigue Exponent* (m)
[1.975,7.478]	[2.145,8.122]	2.342

The Interval S-N relationship determined from these parameters is plotted in Figure 2.

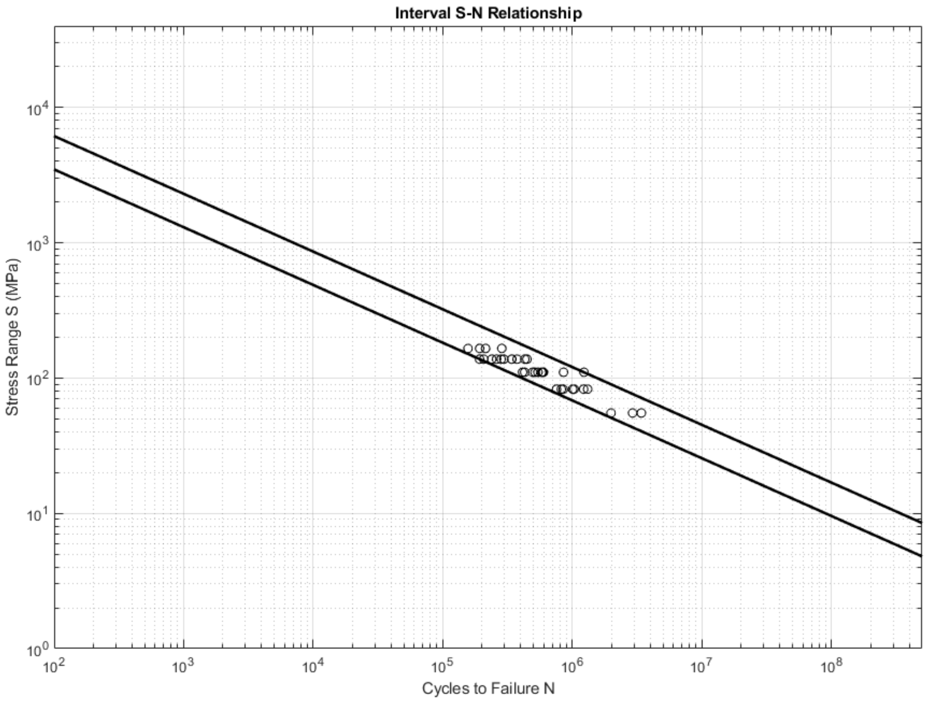

Figure 2. *Interval S-N relationship for an A514 steel girder with cover plates, un-welded ends*

Step 2. The interval stress ranges $\boldsymbol{S_i}$ are determined and enumerated in Table IV.

Table IV. Interval stress ranges

Interval Stress Range ($\boldsymbol{S_i}$) MPa	(ksi)
[6.83,6.96]	[0.99,1.01]
[10.24,10.45]	[1.49,1.52]
[13.65,13.93]	[1.98,2.02]
[17.06,17.41]	[2.48,2.53]
[20.48,20.89]	[2.97,3.03]
[23.89,24.37]	[3.47,3.54]
[27.30,27.85]	[3.96,4.04]
[30.72,31.34]	[4.46,4.55]
[34.13,34.82]	[4.95,5.05]
[37.54,38.30]	[5.45,5.56]

Step 3. The interval damage occurring during the one day data collection period ($t_d = 1/365\ years$) is calculated as: $\boldsymbol{D_d} = [1.640, 6.504] \times 10^{-5}$.

Step 4. The values for interval mean damage, interval existing damage ($t_a = 20\ years$), interval fatigue life, and interval remaining life are determined and tabulated (Table 5).

4.3. VERIFICATION

To verify the values and sharpness of the obtained interval bounds, a Monte Carlo simulation is conducted using 10^8 realizations of uniformly distributed random variables. The MCS results are compared to the interval results in Table V.

Table V. Fatigue damage and life prediction results

Parameter	*Interval Results*	*MCS Results*
Interval mean damage ($\boldsymbol{D_m}$)	[0.0060, 0.0237]	[0.0060,0.0236]
Interval existing damage ($\boldsymbol{D_e}$)	[0.1197, 0.4748]	[0.1204,0.4717]
Interval fatigue life ($\boldsymbol{t_l}$) (years)	[42.12, 167.11]	[42.40,166.08]
Interval remaining life ($\boldsymbol{t_r}$) (years)	[22.12, 147.11]	[22.40,146.08]

4.4. OBSERVATIONS

The results depicted in Table 5 shows that the lower bound (conservative) prediction for the remaining life of the bridge is 22.12 years while its upper bound is 147.11 years. This existence of large variation in the remaining fatigue life attests to the hyper sensitivity of fatigue analysis to the presence of uncertainties. This means that large uncertainties in the laboratory or field tests will result in wide uncertainties for the prediction of the remaining fatigue life.

Moreover, and as expected, the results obtained by Monte Carlo simulation are inner bounds of those obtained by the developed method. Furthermore, the interval results are sharp, which can be attributed to the independence of the interval variables in the entire computational scheme.

5. Summary and Conclusions

In this paper, a method to consider uncertainties in fatigue life prediction using interval variables is developed. This method considers uncertainties present in both laboratory and field data as well as uncertainties introduced by the analysis process. Because of its set-based approach, this method computes sharp bounds on the existing damage and remaining fatigue life of the structure with minimal computation effort. The interval results and their sharpness obtained by this method are verified by the Monte Carlo simulations performed. The analysis shows the hyper sensitivity of fatigue analysis to the presence of uncertainties which results in large variations in the fatigue life prediction. Therefore, it is of utmost importance to minimize the errors and uncertainties in the input of the analysis process. The method's simplicity and versatility make it an attractive option to consider uncertainties in the remaining fatigue life problem, especially when limited data is available.

References

Ang, A. H.-S., and W. H. Munse. “Practical reliability basis for structural fatigue.” Meeting preprint 2494, ASCE Annual National Structural Engineering Conference. New Orleans, April 1975: American Society of Civil Engineers. (1975).

Basquin, O.H. “The exponential law of endurance tests”. American Society of Testing Materials, 10: 625-630. (1910).

Diekfuss, J.A. and C.M. Foley. “Modeling Error Uncertainty Characterization for Reliability-Based Fatigue Assessment in Sign Support Structures”. Journal of Structural Engineering, 124(7): 04016042, (2016).

Fisher, J.W., K.H. Frank, M.A. Hirt, and B.M. McNamee. “Effect of weldments on the fatigue strength of steel beams”. Fritz Engineering Laboratory Report No. 334.2. (1969).

Gu, Z. and X. Ma. “A feasible method for the estimation of the interval bounds based on limited strain-life fatigue data”. International Journal of Fatigue, 116: 172-179, (2018).

Hahin, C., J.M. South, J. Mohammadi, and R.K. Polepeddi. “Accurate and Rapid Determination of Fatigue Damage in Steel Bridges”, Journal of Structural Engineering, 119(1), pp 150-168. (1993).

Heimlich, O., Nehmeier M., and Wolff von Gudenberg, J. “Variants of the general interval power function”. Soft Comput, 17:1357-1366, (2013).

Kwon, K., D.M. Frangopol, and M. Soliman. “Probabilistic Fatigue Life Estimation of Steel Bridges by Using a Bilinear S-N Approach”. Journal of Bridge Engineering, 17(1): 58-70, (2012).

Long, X.Y., C. Jiang, K. Liu, X. Han, W. Gao, and B.C. Li. “An interval analysis method for fatigue crack growth life prediction with uncertainty”. Computers and Structures, 210: 1-11, (2018).

Matsuishi, M., and T. Endo. "Fatigue of metals subjected to varying stress". Japan Society of Mechanical Engineering. (1968).

Miner, M.A. “Cumulative Damage in Fatigue”. Journal of Applied Mechanics, 12: A159-A164. (1945).

Moore, R.E. Interval Analysis. Prentice-Hall, Englewood Cliff, NJ, (1966).

Sen, T.K. “Probability of Fatigue Failure in Steel Catenary Risers in Deep Water”. Journal of Engineering Mechanics, 132(9): 1001-1006, (2006).

Sofi, A., G. Muscolino, and F. Giunta. “Fatigue analysis of structures with interval axial stiffness subjected to stationary stochastic excitations”. Meccanica, 54: 1471-1487, (2019).