Interval analysis of the forced vibration of beams with uncertain damage

F. Cannizzaro[1], N. Impollonia[1], G. Cocuzza Avellino[1], S. Caddemi[1], and I. Caliò[1]

[1]Department of Civil Engineering and Architecture, University of Catania, Catania 95125, Italy, francesco.cannizzaro@dica.unict.it, nicola.impollonia@unict.it, giuseppecocuzzaavellino@gmail.com, salvatore.caddemi@unict.it, icalio@dica.unict.it

Keywords: *Dynamic response, Generalised functions, concentrated damage, Sherman-Morrison formula, interval analysis.*

Abstract

The presence of concentrated damage in beam like structures implies significant changes in the dynamic properties, and may lead to a vibrating response difficult to predict. In presence of concentrated damage, a reliable macroscopic model of cracks is based on the introduction of internal equivalent rotational springs. Under the hypotheses of non-propagating cracks and no occurrence of the closing crack phenomenon the analysis is restricted to the linear range. Even in this case, analyses are usually conducted either by means of finite element discretization or else by making use of the continuous beam model between two successive cracks requiring the enforcement of continuity conditions at the cracked cross sections. Alternatively, in this study, the local reduction of concentrated cracks is modelled by means of the use of generalised functions (distributions) that allows the formulation of closed form expressions of the mode shapes and the relevant frequency equation for beams in presence of an arbitrary number of concentrated cracks. Even though the adoption of distributions has been proved very effective in the deterministic dynamic analysis of beams, analogous investigation of the distributional model has not been conducted in the more realistic case of cracks affected by uncertainties. The uncertainties in the position and intensity of damage imply the difficulty in adopting for practical purposes deterministic analyses; on the other hand, a powerful possibility to perform parametric analyses is associated to explicit formulations when they are available. In this paper, approximated explicit expressions of the main modal parameters related to the crack severities are provided and the reliability of the proposed formulas are verified with the exact solutions. The proposed methodology is here adopted to assess the variability of the dynamic response of cracked beams due to uncertainties in the crack intensity. More precisely, cracks are introduced with uncertain, but bounded, depths assumed as interval variables; upper and lower bounds of the response of damaged beams with multiple cracks subjected to deterministic loads are evaluated by making use of both time and frequency domain analyses. The efficacy of the proposed approach is corroborated by numerical examples on simple damaged straight beams and allows future advancements for similar analyses on damaged frames in presence of crack uncertainties.