Forward interval propagation through the discrete Fourier transform

Marco De Angelis[1)], **Marco Behrendt**[1,2)], **Liam Comerford** [1)], **Yuanjin Zhang** [3)], **Michael Beer** [1,2)]

[1)]Institute for Risk and Uncertainty, University of Liverpool, UK
[2)]Institute for Risk and Reliability, University of Hannover, Germany
[3)]Engineering Reliability and Stochastic Mechanics, Tongji University, China
{*marco.de-angelis, marco.behrendt*} @*liverpool.ac.uk*

Keywords: *Complex intervals; Dependency tracking; Convex hull; Interval algorithm.*

Abstract

The discrete Fourier transform (DFT) is ubiquitous in signal processing and in engineering computing in general. DFT allows signals to be decomposed into single harmonics and so facilitate the data compression and analysis. The versatility of DFT resides in its unique mathematical property of being linear and invertible. The latter property enables signals to be transformed into the Fourier domain and then back transformed to the original domain with controllable error. The DFT range of applications, from spectral analysis, image processing to differential equations and probability propagation, seems limitless in practice. The DFT is used daily in engineering dynamics to study the harmonic properties, like resonance and damping, of structural response subject to dynamic excitation.

One aspect of the DFT is that it is not inherently suited to deal with imprecise signals, i.e. signal carrying error tolerance or epistemic uncertainty in each individual value of the sequence being transformed. This is the case, for example, when a time signal is acquired with cheap sensors with limited precision, or with weary sensors affected by faults or unavailability. There are innumerable cases where the signal can be inaccurate and uncertain, and however informative in sensing critical circumstances (Zhang et al.). It is therefore of great advantage being able to rigorously analyse in the Fourier domain imprecise signals without loss of information.

This paper tackles the problem of rigorous forward error propagation in DFT. We propose an algorithm for interval-valued signals, which makes neither distributional nor dependence assumptions between values in the provided bounds. The proposed algorithm yields best-possible bounds on the amplitude of the DFT for real and complex valued sequences. We show that computing the exact bounds for the amplitude of the DFT can be achieved with an exhaustive examination of all possible corners in the convex hull set of points of the interval-shaped domain.

References

Zhang, Y., Comerford, L., Kougioumtzoglou, I. A., Patelli, E., Beer, M. Uncertainty quantification of power spectrum and spectral moments estimates subject to missing data. *ASCE-ASME Journ. of Risk and Uncert. in Engineering Syst., Part A: Civil Engineering.*