Why Ellipsoids in Mechanical Analysis of Wood Structures

F. N. Schietzold[1)], **J. Urenda**[2,3)], **V. Kreinovich**[3)], **W. Graf**[1)], and **M. Kaliske**[1)]

[1)]Institute for Structural Analysis, Technische Universität Dresden
01062 Dresden, Germany, {niklas.schietzold; wolfgang.graf; michael.kaliske}@tu-dresden.de

Departments of [2)]Mathematical Sciences and [3)]Computer Science
University of Texas at El Paso
El Paso, TX 79968, USA, {jcurenda; vladik}@utep.edu

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Abstract

Wood is a very mechanically anisotropic material. At each point on the wooden beam, the local mechanical properties corresponding to a certain direction depend, e.g., on whether this direction is longitudinal, radial or tangential with respect to the grain orientation of the original tree. This anisotropy can be described in geometric terms, if we select a point x and form *iso-correlation* surfaces – i.e., surfaces formed by points y with the same level of correlation $\rho(x, y)$ between local changes in the vicinities of the points x and y.

Empirical analysis shows that for each point x, the corresponding surfaces are well approximated by concentric homothetic ellipsoids; see, e.g., Schietzold (2019). In this paper, we provide a theoretical explanation for this empirical fact. The main ideas behind our explanation are similar to the ideas used in Finkelstein, Kosheleva, and Kreinovich (1996); Li, Ogura, and Kreinovich (2002) to explain efficiency of ellipsoid approximation in numerical analysis; the main difference is that now we consider *not* classes of sets (such as the class of all ellipsoids), but classes of *families* of sets (e.g., the class of all families of concentric homothetic ellipsoids). Specifically, we show that for the smallest dimension d for which it is possible to have an affine-invariant optimality criterion on the space of all such d-dimensional classes, for any such criterion, the optimal family consists of concentric homothetic ellipsoids. Thus, such families of ellipsoids provide the optimal approximation to the actual surfaces – at least in the *first* approximation, i.e., approximation corresponding to the smallest possible number of parameters.

References

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