Random set solutions to partial differential equations

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Abstract

In many branches of engineering, such as linear elasticity or wave propagation, spatial randomness of the underlying material or medium is described by random fields, which enter as coefficients in the equilibrium equations or the equations of motion. Prototypical equations are $-\mathrm{div}\,(a(x)\,\mathrm{grad}\,u(x)) = f(x)$ for the transversal displacement $u(x)$ of a membrane or $\partial_t v(t,x) + a(x)\cdot \mathrm{grad}\,v(t,x) = 0$ for the transport of a quantity $v(t,x)$ in a random environment. Here $a(x)$ is a random field encoding the probabilistic uncertainty of the material properties. Further epistemic uncertainty can be entered by modelling the statistical parameters of the random field (mean, variance, correlation length) as intervals. Consequently, the solution is a set-valued random field or a set-valued stochastic process.

A mathematical framework for combining probability and intervals is the theory of random sets. In this approach, the solution is viewed as a random set with values in a function space. This situation has been investigated from several viewpoints, including computational aspects, in the literature; see e.g. Sofi (2015); Oberguggenberger (2015); Do, Gao, Song and Beer (2016); Oberguggenberger and Wurzer (2019); Sofi, Muscolina and Giunta (2020). The mathematical set-up involves a number of subtleties. In order to establish that the solution is a properly defined random set, the main issue is the continuous dependence of the solution on the parameters of the random field. Another issue is the incorporation of interval parameters in the generation of the random set.

In this talk, the mathematical modelling questions will be highlighted and exemplified.

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