Elucidating appealing features of differentiable autocorrelation functions: a study on the modified exponential kernel

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Abstract

Research on stochastic processes in recent decades has pointed out that, in the context of modelling spatial or temporal uncertainties, auto-correlation functions that are differentiable at the origin have advantages over functions that are not differentiable. For instance, the non-differentiability of e.g., single exponential autocorrelation functions yields non-smooth sample paths. Such sample paths might not be physically possible or might yield issues when used as random parameters in partial differential equations (such as encountered in e.g., mechanical equilibrium problems). Further, it is known that due to the non-differentiability of certain auto-correlation functions, more terms are required in the stochastic series expansion representations. This makes these representations less efficient from a computational standpoint.

This paper elucidates some additional appealing features of autocorrelation functions which are differentiable at the origin. Further, it focusses on deepening the argumentations in favor of these functions already available in literature. Specifically, focus is placed on single exponential, modified exponential and squared exponential autocorrelation functions. To start, it is shown that the power spectrum of differentiable kernels converges faster to zero wiht increasing frequency as compared to non-differentiable ones. This property allows capturing the same amount of energy of the spectrum with a smaller cut-off frequency, and hence, less stochastic terms. Further, this point is examined with reqards to the Karhunen-Loève series expansion and first and second order Markov processes, generated via auto-regressive representations.