

Interval Matrix Multiplication Using Fast Low-precision Arithmetic on GPU

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Abstract

This research concerns interval arithmetic, especially, we focus on interval matrix multiplication using the Graphics Processing Unit (GPU). Interval matrix multiplication has an important role in numerical linear algebra. There are several fast methods for interval matrix multiplication. See Rump (1999, 2012); Ogita and Oishi (2005); Ozaki et al. (2012, 2015). The points at issue are the tightness of computed intervals and the cost of methods.

Single precision arithmetic (binary32 in IEEE 754) is performed very fast compared to double precision arithmetic (binary64 in IEEE 754) on some GPUs, for example, GeForce GTX series produced by NVIDIA. We exploit fast binary32 arithmetic and develop a method with an adaptive cost for interval matrix multiplication based on error-free transformations by Ozaki et al. (2012). We will show the performance of the proposed method using GeForce GTX 2070 by NVIDIA and its Tensor Cores. Finally, we introduce an application of the proposed method to computer-assisted proof.

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